

**NOVEMBER 2015**

**DR. Z's CORNER**

***Conquering the FE & PE exams***  
***Examples & Applications***

**Topics covered in this month's column:**

- FE CIVIL Exam Topics & Number of Questions
- Technology Usage (Casio fx-115 ES *PLUS*)
- Math, Angle Conversions, From DMS to Decimal and Vice Versa
- Probability & Statistics (Manual and Calculator)
- Math, Binary and Decimal Conversions
- Absolute Error, Relative Error Computations
- Transportation, Horizontal Curves
- Bearings and Azimuth Computations
- Centroids & Moments of Inertia
- Plane Stress & Transformation of Plane Stress
- Cantilever Retaining Walls

# **FUNDAMENTALS OF ENGINEERING**

## **CIVIL EXAM TOPICS**

### **Computer-Based Test (CBT)**

**Total Number of Questions: 110**

**Time: 6 hours**

The new Civil FE Computer-Based Test (CBT) consists of 110 multiple-choice questions (Each problem only one question) the examinee will have 6 hours to complete the test.

- **Mathematics (Approx. 9 questions\*)**
- **Probability and Statistics (5 questions)**
- **Computational Tools (5 questions)**
- **Ethics and Professional Practice (5 questions)**
- **Engineering Economics (5 questions)**
- **Statics (9 questions)**
- **Dynamics (5 questions)**
- **Mechanics of Materials (9 questions)**
- **Civil Engineering Materials (5 questions)**
- **Fluid Mechanics (5 questions)**
- **Hydraulics and Hydrologic Systems (10 questions)**
- **Structural Analysis ( 8 questions)**
- **Structural Design ( 8 questions)**
- **Geotechnical Engineering ( 12 questions)**
- **Transportation Engineering ( 10 questions)**
- **Environmental Engineering ( 8 questions)**

\* Here the number of questions are the average values taken from the NCEES Reference Handbook (Version 9.3 / Computer-Based Test)

# CASIO *fx-115 ES PLUS*



## FX-115ESPLUS

This \$17 gadget will definitely be your best helper during your FE or PE exams. The Casio fx-115 is the most popular type of calculator for the exams. It is Casio's most advanced scientific calculator that features new Natural Textbook Display and Improved Math Functionality.

It is a good idea to use your CASIO fx-115 ES *PLUS* all the time when you are solving problems. Only then you can get used to the calculator.

# IMPORTANT FEATURES OF CASIO fx-115ES PLUS

The fx-115ES PLUS retains the Natural-V.P.A.M. (Visually Perfect Algebraic Method), which means that you can enter equations and expressions exactly as written.

**When you hit SHIFT MODE, you'll get the following screen:**

```
1:MthIO 2:LineIO
3:Deg   4:Rad
5:Gra   6:Fix
7:Sci   8:Norm
```

The **MthIO** method allows the user to enter equations and expressions exactly as written. Fractions, radicals, expressions with  $\pi$ , and calculus function templates are all there. Exact answers can include fractions, square roots, and coefficients of  $\pi$  (MthIO).

One-line entry method, the **LineIO** method is also available. Many students prefer the **MathIO** method.

## Some important features of CASIO fx-115ES PLUS

- \* Integrals of  $f(x)$
- \* Numeric Derivatives:  $d/dx$
- \* Sums of a function:  $\sum f(x)$
- \* **Base modes** Decimal, Octal, Binary, Hexadecimal
- \* **Numeric solver**, of equations and roots of expressions.
- \* **CALC** button allows for calculating expressions repeated amount of times.
- \* **Statistics** including 1-variable, linear regression ( $a+bx$ ), quadratic regression ( $a+bx+cx^2$ ), cubic regression ( $a+bx+cx^2+dx^3$ ), 2 types of exponential ( $a + b * e^x$  and  $a x^b$ ), power ( $b a^x$ ), logarithmic ( $a + b \ln x$ ), and inverse ( $a + b/x$ ).

\* **Equations** - 2x2 and 3x3 simultaneous equations, quadratic, and cubic equations

\* **Matrices**: functions include transpose, inverse, and determinant

\* **Vectors**

\* Multi Line Statements with the colon (:)

\* Complex Number Mode

**How to clear calculator history, while in COMP mode?**

press the [ ON ] key.

**How to force approximate answers?**

press [SHIFT] [ = ].

\* The total number of available memories are 9.

(A, B, C, D, X, Y, M, E, F).

\* You have the ability to calculate using repeated numbers. For example, you can type the decimal form of  $1/3$  using 0.3 with the bar above the three.

\* New number functions are: GCD, LCM, Integer Part, Fractional Part, Random Integers, Integer Division ( $\div R$ ) that gives quotient and remainder, and Prime Factorization (up to three digit factors).

\* To factor a number, enter it, press [ = ], then [SHIFT], [  $\circ$  ' " ].

\* Products of function  $f(x)$ :  $\prod f(x)$ .

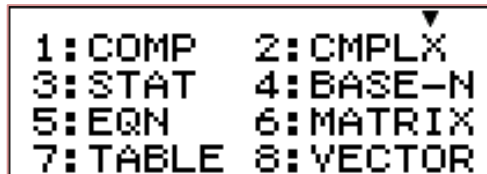
\* In Table Mode you can include two functions  $f(x)$  and  $g(x)$ . This is turned on in SET UP.

\* The [ALPHA] key acts as both as a variable key and a second shift key.

\* The rref and ref functions are added to the Matrix Mode (but not eigenvalues).

## CALCULATOR MODES

The Casio fx-115 has eight "modes." These modes are accessed by pressing the [ **MODE** ] button.



Here are the 8 modes of Casio fx-115:

- |                  |  |
|------------------|--|
| 1: <b>COMP</b>   | computation mode (for most calculations) |
| 2: <b>CMPLX</b>  | complex mode                             |
| 3: <b>STAT</b>   | statistics mode                          |
| 4: <b>BASE-N</b> | base-N mode                              |
| 5: <b>EQN</b>    | equation mode                            |
| 6: <b>MATRIX</b> | matrix mode                              |
| 7: <b>TABLE</b>  | table mode                               |
| 8: <b>VECTOR</b> | vector mode                              |

How to set your calculator into computation mode?

[ **MODE** ] [ **1** ]

How to set your calculator into statistics mode?

[ **MODE** ] [ **3** ]

## STATISTICS MODE IN CASIO fx-115ES PLUS

First press the [ **MODE** ] button and you will get the following screen:

1:COMP	2:CMPLX
3:STAT	4:BASE-N
5:EQN	6:MATRIX
7:TABLE	8:VECTOR

Then choose option 3 and you will see the following screen:

1:1-VAR	2:A+BX
3:Y+CX <sup>2</sup>	4:ln X
5:e <sup>X</sup>	6:A·B <sup>X</sup>
7:A·X <sup>B</sup>	8:1/X

Let's give an example using all the above steps.

## FUNDAMENTALS OF ENGINEERING (FE)

### PROBABILITY AND STATISTICS

FULL SOLUTION  
NEXT PAGE

#### Problem:

A data set is given as listed below:

8, 25, 7, 5, 8, 3, 10, 12, 9

(1) The **mean** of this set is most nearly:

- (A) 7.98
- (B) 8.15
- (C) 9.67
- (D) 12.85

(2) The **standard deviation** is most nearly:

- (A) 6.32
- (B) 7.85
- (C) 8.25
- (D) 9.14



**Problem:**

8, 25, 7, 5, 8, 3, 10, 12, 9

Consider the data set given above:

- (a) Calculate the mean ( $\bar{y}$ )
- (b) Calculate the Standard Deviation ( $s_y$ )

**Solution:**

The **mean** is the sum of scores divided by n where n is the number of scores.

$$1. \bar{y} = \frac{\sum y}{n} = (8+25+7+5+8+3+10+12+9)/9$$

$$= 9.67$$

2. the standard deviation may be calculated using the following formula:

$$s_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}} \quad \text{Deviation} = (y_i - \bar{y})$$

In order to calculate the values in the standard deviation formula, the following table may be used:

Deviation = Score - Mean
--------------------------

Score	Mean	Deviation	(Deviation) <sup>2</sup>
-----			
8	9.67	- 1.67	2.79
25	9.67	+15.33	235.01
7	9.67	- 2.67	7.13
5	9.67	- 4.67	21.81
8	9.67	- 1.67	2.79
3	9.67	- 6.67	44.49
10	9.67	+ .33	.11
12	9.67	+ 2.33	5.43
9	9.67	- .67	.45

$$\Sigma = 320.01$$

## Standard Deviation( $S_y$ )

$$s_y = \sqrt{\frac{\sum (y_i - y)^2}{n-1}} = \sqrt{\frac{320.01}{9-1}} = 6.32$$

$S_y = 6.32$
--------------

Alternate method for calculating the Standard Deviation:  
(The Raw Score Method)

Consider the raw scores 8, 25, 7, 5, 8, 3, 10, 12, 9.

1. First, square each of the scores.
2. Determine n, which is the number of scores.
3. Compute the sum of  $y_i$  and the sum of  $y_i^2$
4. Then, calculate the standard deviation as illustrated below.

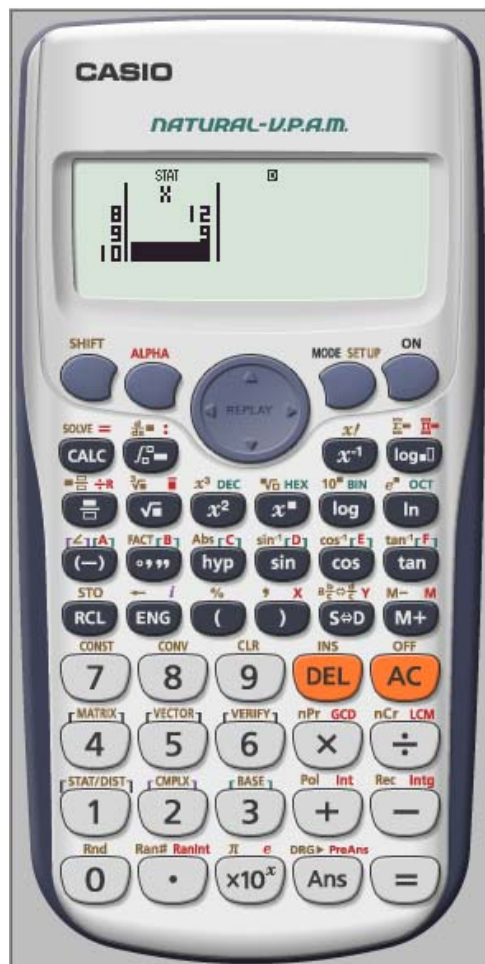
score( $y_i$ )	$y_i^2$	
8	64	
25	625	
7	49	$n = 9$
5	25	
8	64	$\sum y_i = 87$
3	9	
10	100	$\sum y_i^2 = 1161$
12	144	
9	81	
---	----	
87	1161	

## Standard Deviation( $S_y$ )

$$s_y = \sqrt{\frac{\sum y_i^2 - (\sum y_i)^2 / n}{n-1}} = \text{square root}[(1161) - (87*87)/9)/(9-1)]$$
$$= \text{square root}[(1161 - (7569/9))/8] = 6.32$$

**In *FE* exam CASIO fx-115 SE PLUS should be used**

# PROBABILITY & STATISTICS WITH CASIO fx-115 ES PLUS



1:Type	2:Data
3:Sum	4:Var
5:Distr	6:MinMax

1:n	2: $\bar{x}$
3:σx	4:sx

STAT	0
$\bar{x}$	
	9.666666667

STAT	0
sx	
	6.32455532

MODE 3 1 8 = 2 5 = 7 = 5 = 8 = 3

= 1 0 = 1 2 = 9 =

AC SHIFT 1 4 2 =

AC SHIFT 1 4 4 =

## CASIO / fx-115 ES PLUS

**MODE**

- (1) **COMP**
- (2) **CMPLX**
- (3) **STAT**
- (4) **BASE-N**
- (5) **EQN**
- (6) **MATRIX**
- (7) **TABLE**
- (8) **VECTOR**

**SHIFT**

**SETUP**

- (1) **Mth-IO**
- (2) **Line-IO**
- (3) **Deg**
- (4) **Rad**
- (5) **Gra**
- (6) **Fix**
- (7) **Sci**
- (8) **Norm**

### Frequently asked two Number Systems:

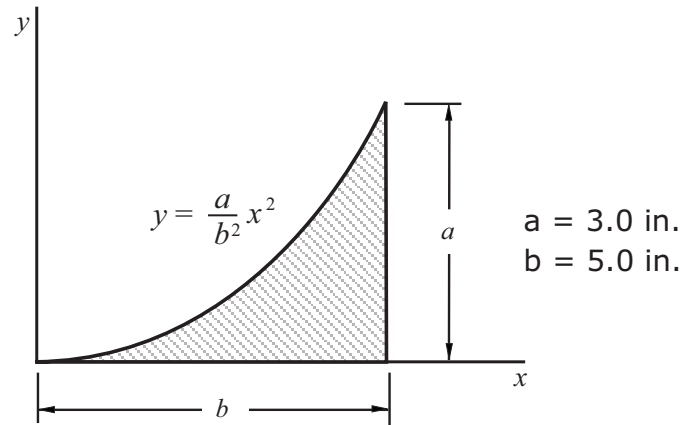
- 1- Decimal Number System (base 10)
- 2- Binary Number System (base 2)

In Decimal System 10 different digits are used to create any number, but in Binary System only 0s and 1s are used to create any number

DECIMAL	BINARY
2	10
3	11
5	101
6	110
8	1000
9	1001
10	1010
12	1100
14	1110
15	1111
19	10011
25	11001

## STATICS / CENTROIDS

### Problem:



The equation and dimensions of an area are given as shown in the figure. Using the listed data answer the following questions:

- (1) the shaded area (in.<sup>2</sup>) is most nearly
  - (A) 3.55
  - (B) 5.00
  - (C) 6.50
  - (D) 7.40
- (2) the distance  $\bar{x}$  (in.) of the centroid is most nearly
  - (A) 3.05
  - (B) 3.15
  - (C) 3.60
  - (D) 3.75
- (3) the distance  $\bar{y}$  (in.) of the centroid is most nearly
  - (A) 0.75
  - (B) 0.90
  - (C) 1.15
  - (D) 1.30

# ANGLE CONVERSIONS FROM (DMS) TO DECIMAL DEGREES MANUAL CALCULATIONS

**Degrees, minutes and seconds: ( $d^{\circ} m' s''$ )**

One degree is equal to 60 minutes and 3600 seconds:

$$1^{\circ} = 60' = 3600''$$

One minute is equal to  $1/60$  degrees:

$$1' = (1/60)^{\circ} = 0.01666667^{\circ}$$

One second is equal to  $1/3600$  degrees:

$$1'' = (1/3600)^{\circ} = 0.000277778^{\circ}$$

For an angle with  $d$  (integer) degrees,  $m$  minutes, and  $s$  seconds:

$$d^{\circ} m' s''$$

The formula to convert DMS to decimal degrees:

$$\text{Angle} = d + m / 60 + s / 3600$$

## Example

Convert 5 degrees 25 minutes and 30 seconds angle to decimal degrees:

$$\text{Angle} = 5^{\circ} 25' 30''$$

The decimal degrees  $dd$  is equal to:

$$\begin{aligned}\text{Angle} &= d + m/60 + s/3600 = 5^{\circ} + 25'/60 + 30''/3600 \\ &= 5.425^{\circ}\end{aligned}$$

MANUAL  
CALCULATION

# ANGLE CONVERSIONS

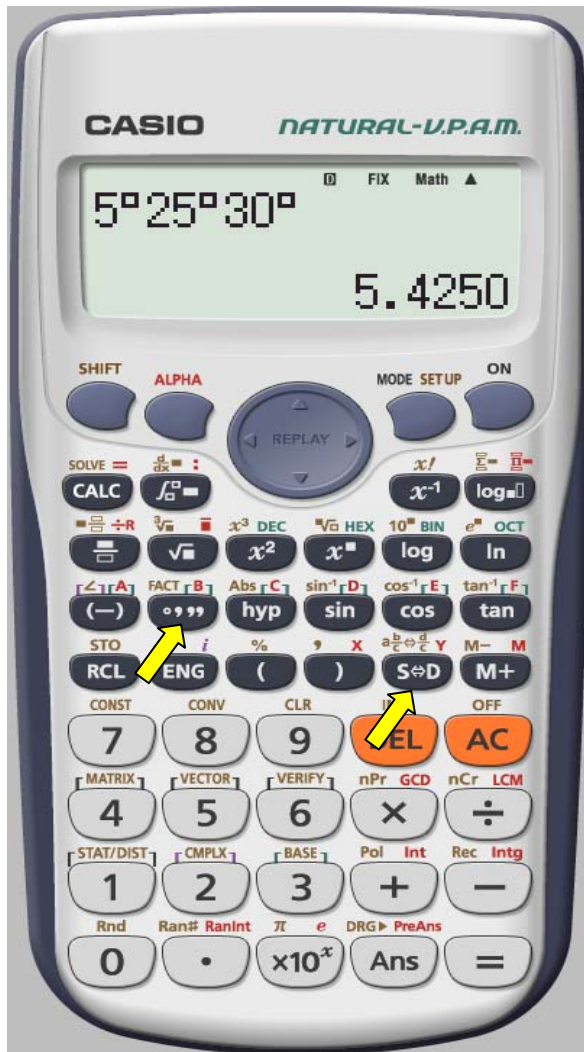
## FROM (DMS) TO DECIMAL DEGREES

### USING CALCULATOR

#### Problem:

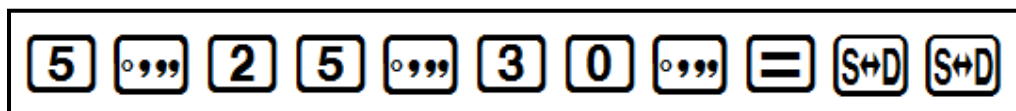
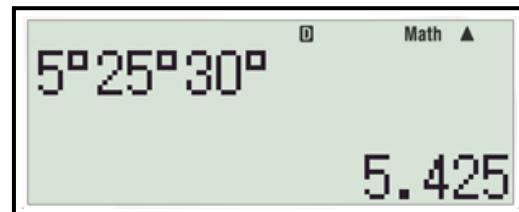
Convert the angle given as 5 degrees, 25 minutes, and 30 seconds to decimal degrees using your calculator.

#### Solution:



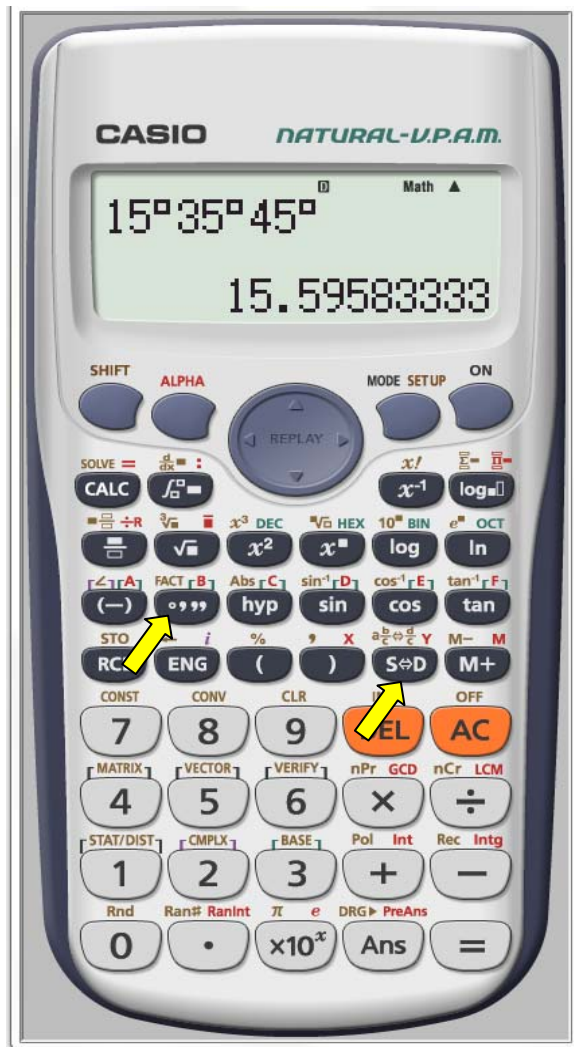
TECHNOLOGY  
USAGE

*Angle = 5°, 25', 30"*



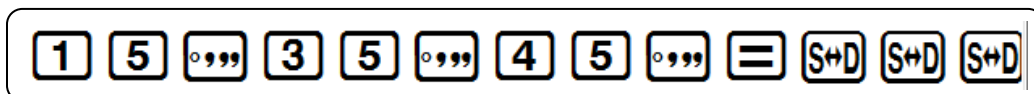
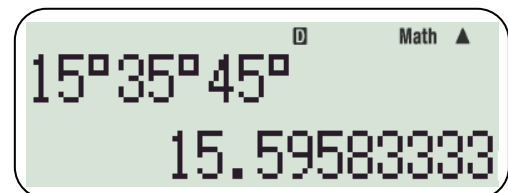
KEY SEQUENCE

# ANGLE CONVERSIONS FROM (DMS) TO DECIMAL DEGREES USING CALCULATOR



Important  
Keys

**Angle =  $15^{\circ}, 35', 45''$**



**KEY SEQUENCE**



## ANGLE CONVERSIONS FROM (DMS) TO DECIMAL DEGREES SUPPLEMENTAL PROBLEMS

(1)

Degrees:	<input type="text" value="15"/>	°
Minutes:	<input type="text" value="35"/>	'
Seconds:	<input type="text" value="45"/>	"

MANUAL  
CALCULATION

Decimal degrees:  $15^{\circ} 35' 45''$   
 $= 15^{\circ} + 35'/60 + 45''/3600$   
 $= 15.59583^{\circ}$

(2)

Degrees:	<input type="text" value="12"/>	°
Minutes:	<input type="text" value="20"/>	'
Seconds:	<input type="text" value="36"/>	"

Decimal degrees:  $12^{\circ} 20' 36''$   
 $= 12^{\circ} + 20'/60 + 36''/3600$   
 $= 12.34333^{\circ}$

# ANGLE CONVERSIONS

## FROM DECIMAL TO DEGREES, MINUTES, SECONDS

### SUPPLEMENTAL PROBLEMS

MANUAL  
CALCULATION

(1)

Decimal degrees:

5.425 °

Degrees, minutes, seconds:

```
d = int(5.425°) = 5°  
m = int((5.425° - 5°) × 60)  
  = 25'  
s = (5.425° - 5° - 25'/60)  
  × 3600 = 30"  
5.425°  
= 5° 25' 30"
```

(2)

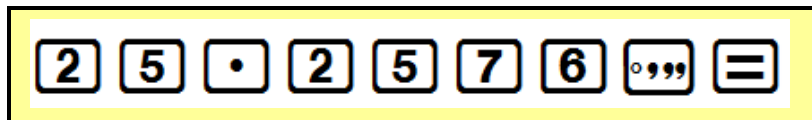
Decimal degrees:

25.2576 °

Degrees, minutes, seconds:

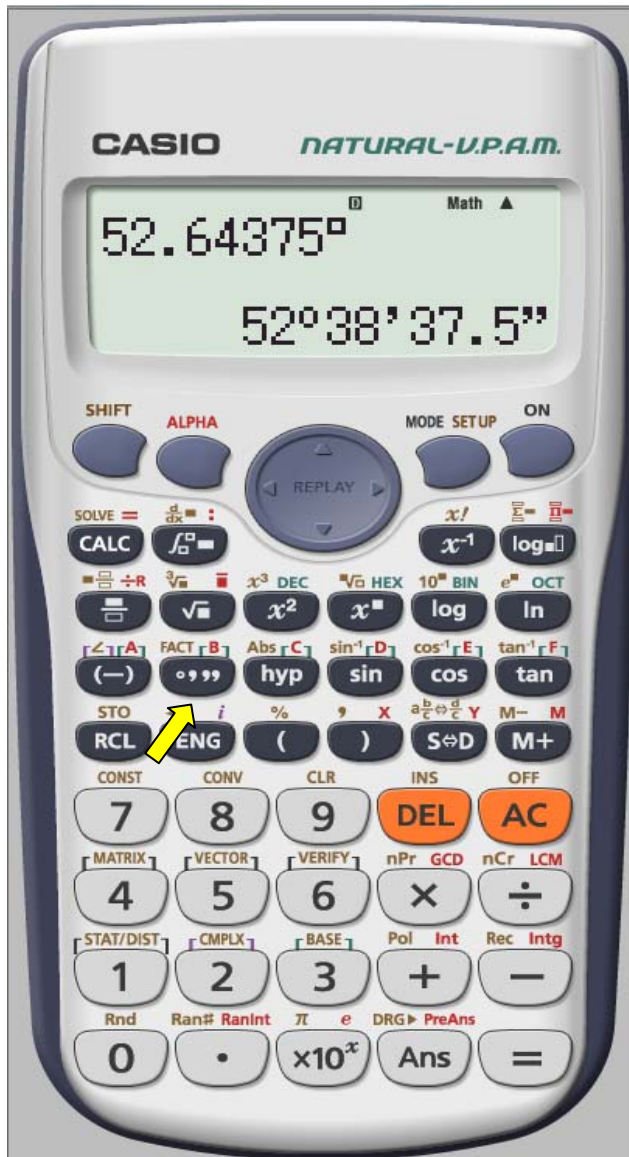
```
d = int(25.2576°) = 25°  
m = int((25.2576° - 25°) ×  
60) = 15'  
s = (25.2576° - 25° -  
15'/60) × 3600 = 27.36"  
25.2576°  
= 25° 15' 27.36"
```

## USING CALCULATOR

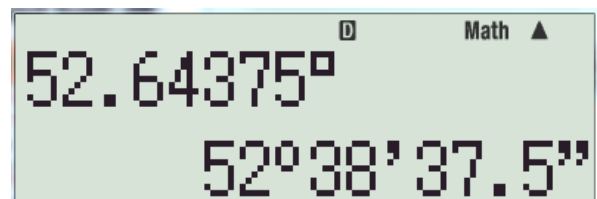


**ANGLE-115-2**  
**ZEYTINCI**  
**FALL 2015**

**ANGLE CONVERSIONS**  
**FROM DECIMAL TO DEGREES, MINUTES, SECONDS**  
**USING CALCULATOR**



TECHNOLOGY  
USAGE



5 2 . 6 4 3 7 5 ° ' ' =

## ABSOLUTE ERROR & RELATIVE ERROR

The accuracy of a computation is very important in numerical analysis.  
There are two ways to express the size of the error in a computed result:

- (a) Absolute Error
- (b) Relative Error

$$\text{ABSOLUTE ERROR} = | \text{True Value} - \text{Approximate Value} |$$

$$\text{RELATIVE ERROR} = \frac{\text{Absolute Error}}{| \text{True Value} |}$$

### Example:

$$\begin{aligned} \text{True Value} &= 10/3 \\ \text{Approximate Value} &= 3.333 \end{aligned}$$

- (a) Determine the absolute error
- (b) Determine the relative error
- (c) Find the significant digits

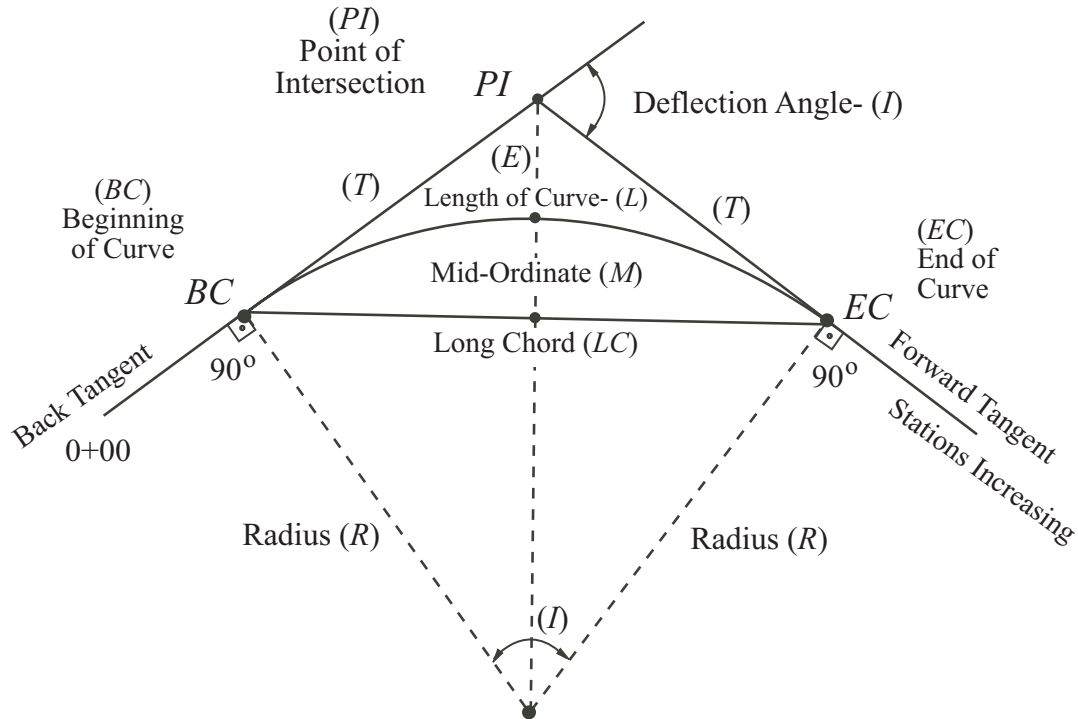
### Solution:

$$\begin{aligned} \text{ABSOLUTE ERROR} &= | \text{True Value} - \text{Approximate Value} | \\ &= 10/3 - 3.333 \\ &= 0.000333... \\ &= 1 / 3000 \end{aligned}$$

$$\begin{aligned} \text{RELATIVE ERROR} &= \frac{\text{Absolute Error}}{| \text{True Value} |} \\ &= \frac{(1/3000)}{(10/3)} \\ &= 1/10,000 \end{aligned}$$

Here, the number of significant digits is 4.

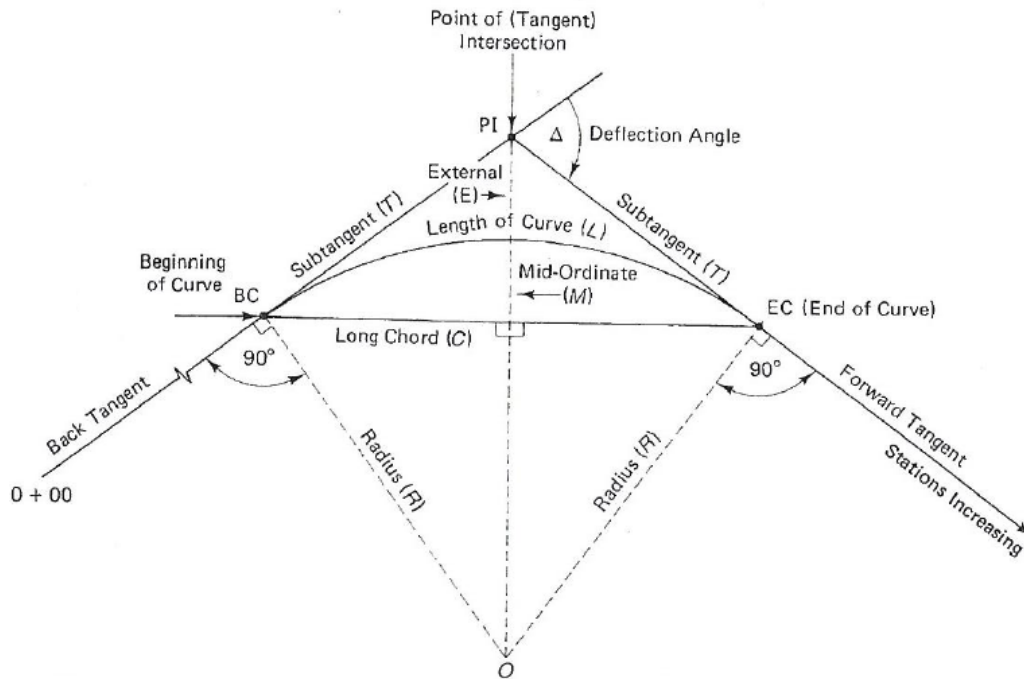
## HORIZONTAL (CIRCULAR) CURVES



- $I$  = Intersection angle (also called  $\Delta$ )
- $D$  = Degree of Curve (Arc Definition)
- $PI$  = Point of Intersection
- $PC$  = Point of Curvature, also called  $(BC)$
- $PT$  = Point of Tangency, also called  $(EC)$
- $L$  = Length of Curve (from  $PC$  to  $PT$ )
- $T$  = Tangent Distance
- $E$  = External Distance
- $R$  = Radius
- $LC$  = Length of Chord
- $M$  = Length of Middle Ordinate

## HORIZONTAL (CIRCULAR) CURVES

### IMPORTANT FORMULAS



$$R = \frac{5729.58}{D}$$

$$R = \frac{LC}{2 \sin(I / 2)}$$

$$T = R \tan(I / 2)$$

$$T = \frac{LC}{2 \cos(I / 2)}$$

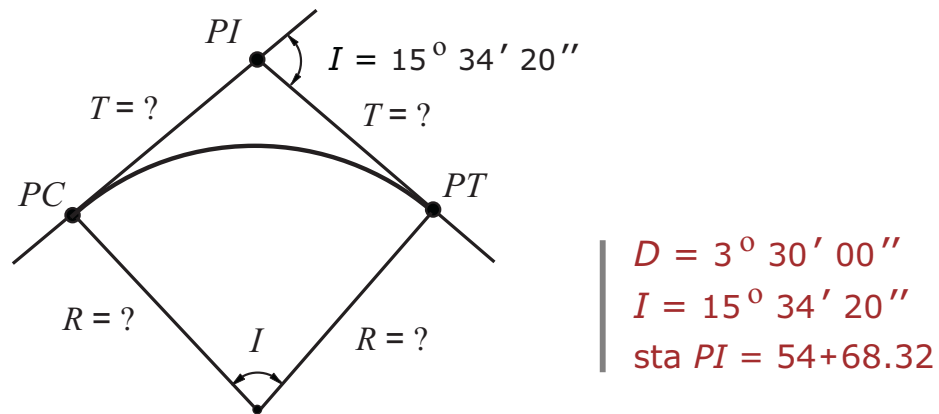
$$L = R I \frac{\pi}{180} = \frac{I}{D} 100$$

$$M = R [1 - \cos(I / 2)]$$

$I$  = Intersection angle (also called  $\Delta$ )  
 $D$  = Degree of Curve (Arc Definition)  
 $PI$  = Point of Intersection  
 $PC$  = Point of Curvature, also called (BC)  
 $PT$  = Point of Tangency, also called (EC)  
 $L$  = Length of Curve (from  $PC$  to  $PT$ )  
 $T$  = Tangent Distance  
 $E$  = External Distance  
 $R$  = Radius  
 $LC$  = Length of Chord  
 $M$  = Length of Middle Ordinate

# TRANSPORTATION

## HORIZONTAL CURVE DESIGN



A  $3^{\circ} 30'$  horizontal curve has forward and backward tangents that intersect at station  $54 + 68.32$ . Knowing that the intersection angle is  $I = 15^{\circ} 34' 20''$ , determine the following:

- (a) the radius of the curve ( $R$ )
- (b) the tangent distance ( $T$ )
- (c) the length of long chord ( $LC$ )
- (d) the external distance ( $E$ )
- (e) the length of middle ordinate ( $M$ )
- (f) the length of curve ( $L$ )
- (g) the station of  $PC$ , sta ( $PC$ )
- (h) the station of  $PT$ , sta ( $PT$ )

NCEES  
 REFERENCE  
 HANDBOOK  
 PAGE-166

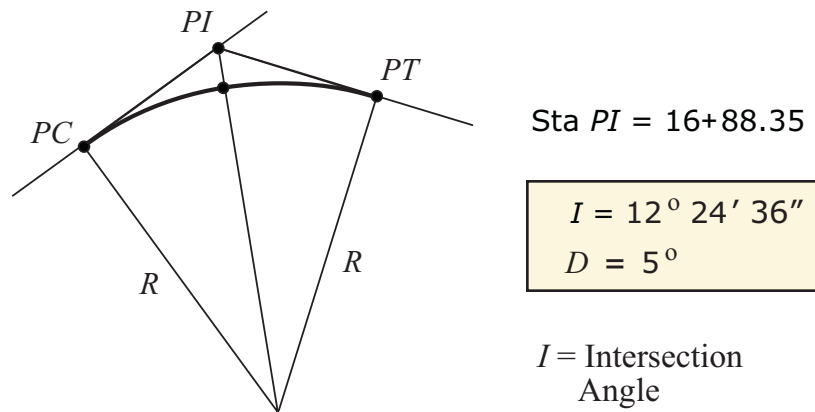
### Answers:

- a) the radius of curve :  $R = 1637.02$  ft.
- (b) the tangent distance :  $T = 223.84$  ft.
- (c) the length of long chord :  $LC = 443.55$  ft.
- (d) the external distance :  $E = 15.23$  ft.
- (e) the length of middle ordinate :  $M = 15.09$  ft.
- (f) the length of curve :  $L = 444.92$  ft.
- (g) the station of  $PC$  : sta ( $PC$ ) :  $52 + 44.48$
- (h) the station of  $PT$  : sta ( $PT$ ) :  $56 + 89.40$



# TRANSPORTATION

## HORIZONTAL CURVES



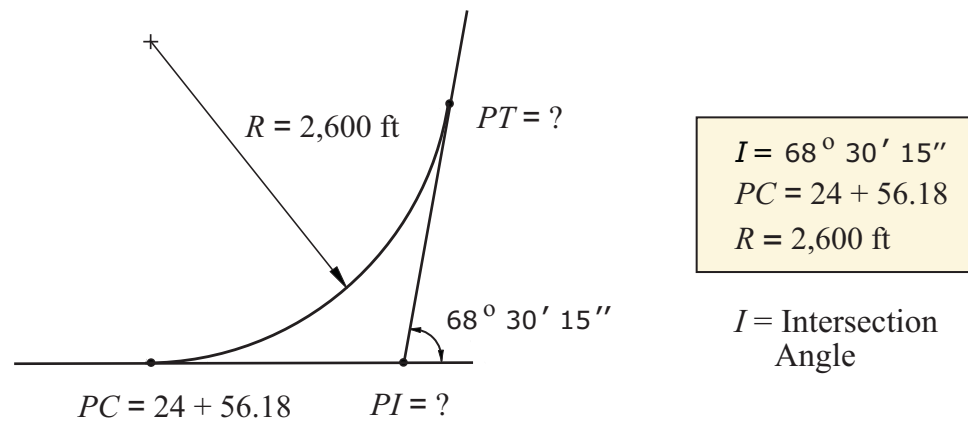
A  $5^\circ$  horizontal curve has forward and back tangents that intersect at station 16+88.35. Knowing that the intersection angle is  $I = 12^\circ 24' 36''$ , answer the following questions:

- (1) the radius (ft) of the curve is most nearly,  $R$ 
  - (A) 1975.60
  - (B) 1694.42
  - (C) 1491.53
  - (D) 1145.92
- (2) the tangent distance (ft) of the curve is most nearly,  $T$ 
  - (A) 247.62
  - (B) 137.54
  - (C) 124.59
  - (D) 101.18
- (3) the station of point of curve -  $PC$  is most nearly,  $PC$ 
  - (A) 10 + 94.65
  - (B) 12 + 85.44
  - (C) 15 + 63.76
  - (D) 17 + 10.15
- (4) the station of point of tangent -  $PT$  is most nearly,  $PT$ 
  - (A) 13 + 94.24
  - (B) 15 + 88.16
  - (C) 16 + 77.82
  - (D) 18 + 11.96

**Important note for students:**

**This problem is for practice and has four questions. In real FE & PE exams, each problem will have ONLY ONE question.**

**Problem:** ( Transportation / Horizontal Curves )

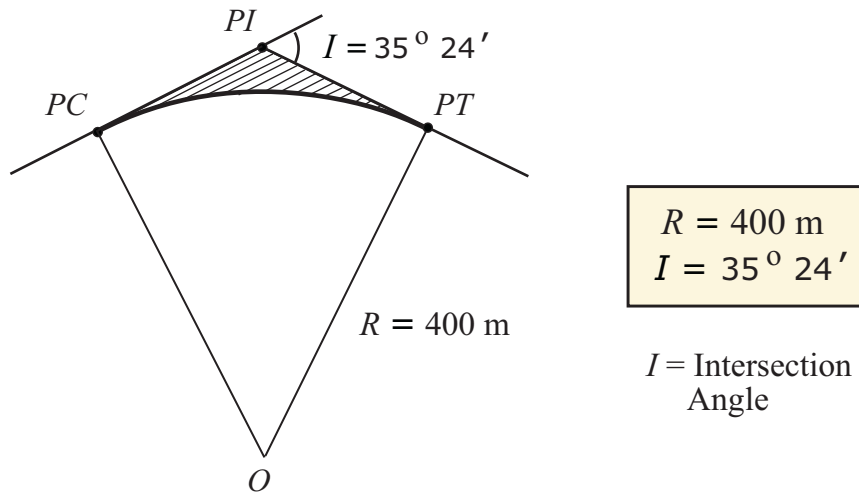


Referring to the horizontal curve shown in the figure above, answer the following questions:

- (1) the tangent distance (ft) of the curve is most nearly,  $T$ 
  - (A) 1575.20
  - (B) 1770.42
  - (C) 1886.36
  - (D) 2026.14
- (2) the external distance (ft) of the curve is most nearly,  $E$ 
  - (A) 545.53
  - (B) 670.42
  - (C) 786.36
  - (D) 826.14
- (3) the station of point of intersection- $PI$  is most nearly,  $PI$ 
  - (A) 32 + 34.55
  - (B) 38 + 42.24
  - (C) 42 + 26.60
  - (D) 45 + 18.47
- (4) the station of point of tangency- $PT$  is most nearly,  $PT$ 
  - (A) 39 + 26.45
  - (B) 43 + 31.16
  - (C) 45 + 16.55
  - (D) 55 + 64.81

## TRANSPORTATION

### HORIZONTAL CURVES

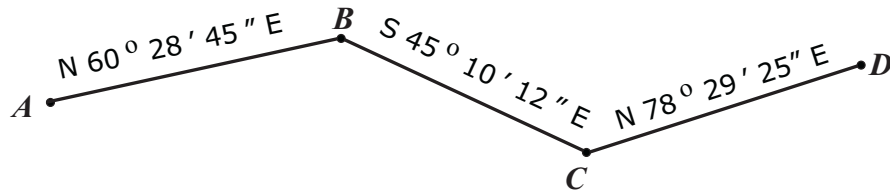


A horizontal curve is given as shown above. The intersection angle and the radius of the curve are given as listed. The shaded area ( $\text{m}^2$ ) between the circular curve and the tangents is most nearly:

- (A) 4,879
- (B) 3,577
- (C) 2,427
- (D) 1,635

## BEARINGS & AZIMUTHS

**Problem:** (Bearings & Azimuths) HK



An open traverse is given as shown in the above figure. Using the given bearings for courses  $AB$ ,  $BC$ , and  $CD$ , answer the following questions:

(1) the north azimuth of  $AB$  (deg-min-sec) is most nearly

- (A)  $33^{\circ} 29' 14''$
- (B)  $49^{\circ} 46' 10''$
- (C)  $51^{\circ} 10' 12''$
- (D)  $60^{\circ} 28' 45''$

(2) the north azimuth of  $BC$  (deg-min-sec) is most nearly

- (A)  $102^{\circ} 21' 34''$
- (B)  $119^{\circ} 46' 10''$
- (C)  $134^{\circ} 49' 48''$
- (D)  $152^{\circ} 36' 38''$

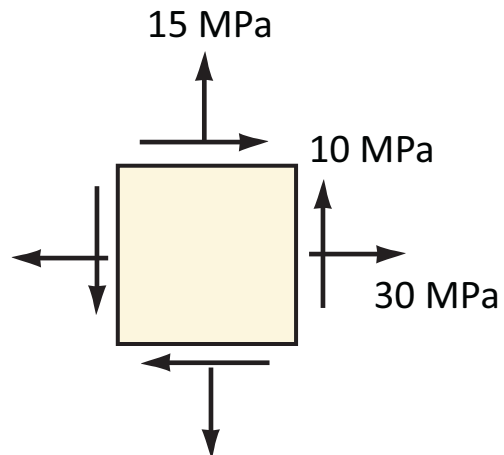
(3) the north azimuth of  $CD$  (deg-min-sec) is most nearly

- (A)  $93^{\circ} 36' 34''$
- (B)  $82^{\circ} 19' 52''$
- (C)  $78^{\circ} 29' 25''$
- (D)  $13^{\circ} 27' 12''$

## MECHANICS OF SOLIDS

### PLANE STRESS

#### Problem:

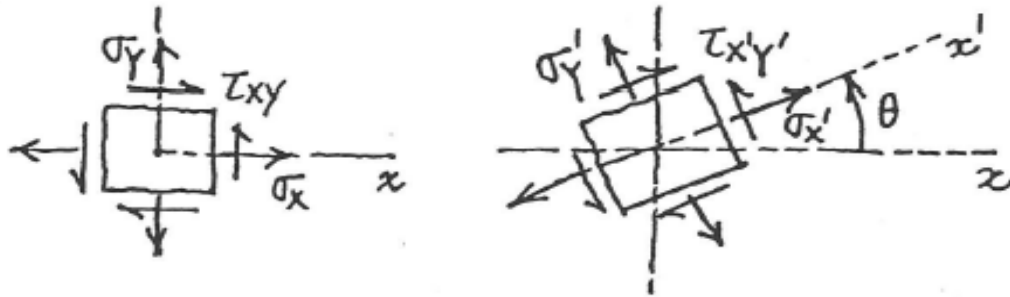


The stresses shown in the figure act at a point in a machine part. Using the given stress element, the maximum inplane shear stress (MPa) in the element is most nearly:

- (A) 26.50
- (B) 22.25
- (C) 15.60
- (D) 12.50

$$\tau_{\max} = ?$$

## TRANSFORMATION OF PLANE STRESS



$$\sigma_{x'} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{max, min} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

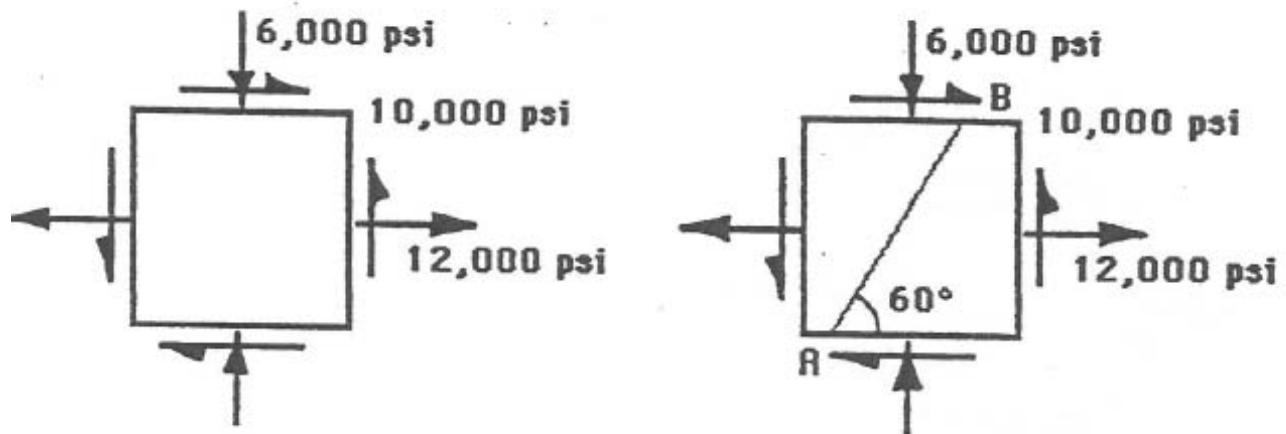
PRINCIPAL PLANES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

PLANES OF MAXI  
SHEARING STRESS.

## TRANSFORMATION OF PLANE STRESS



The stresses shown act at a point in a machine part.  
Find the normal and shearing stresses on plane AB.

$$\begin{aligned} \sigma_x &= +12,000 \text{ psi} = +12 \text{ ksi} \\ \sigma_y &= -6,000 \text{ psi} = -6 \text{ ksi} \\ \tau_{xy} &= +10,000 \text{ psi} = +10 \text{ ksi} \end{aligned}$$

$$\begin{aligned} \theta &= 30^\circ \text{ From vert.} \\ \theta &= -30^\circ \\ \sin 2\theta &= -0.866, \cos 2\theta = 0.5 \end{aligned}$$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

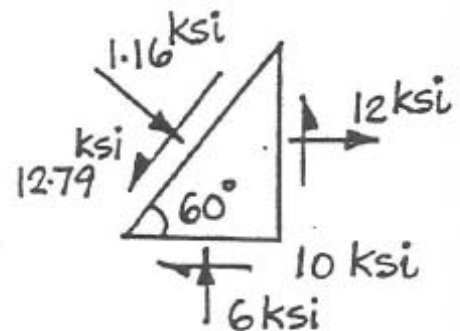
$$\sigma_n = \frac{12 + (-6)}{2} + \frac{(12) - (-6)}{2} (0.5) + 10(-0.866)$$

$$\ast \boxed{\sigma_n = -1.16 \text{ ksi}} \text{ (COMP)}$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_{nt} = -\frac{12 - (-6)}{2} (-0.866) + (10)(0.5)$$

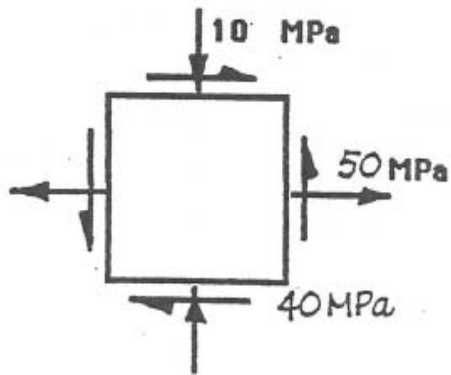
$$\ast \boxed{\tau_{nt} = 12.79 \text{ ksi}}$$



ANS :  $\sigma_n = 1.16 \text{ ksi (Comp)}$

$\tau_{nt} = 12.79 \text{ ksi} \checkmark$

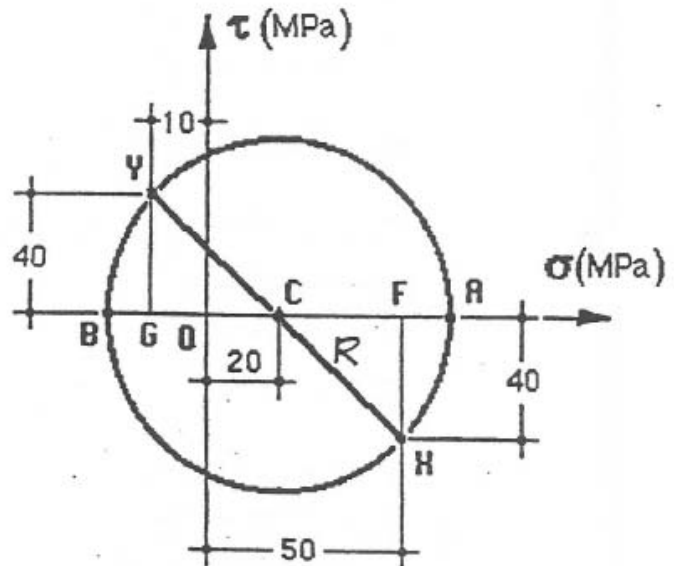
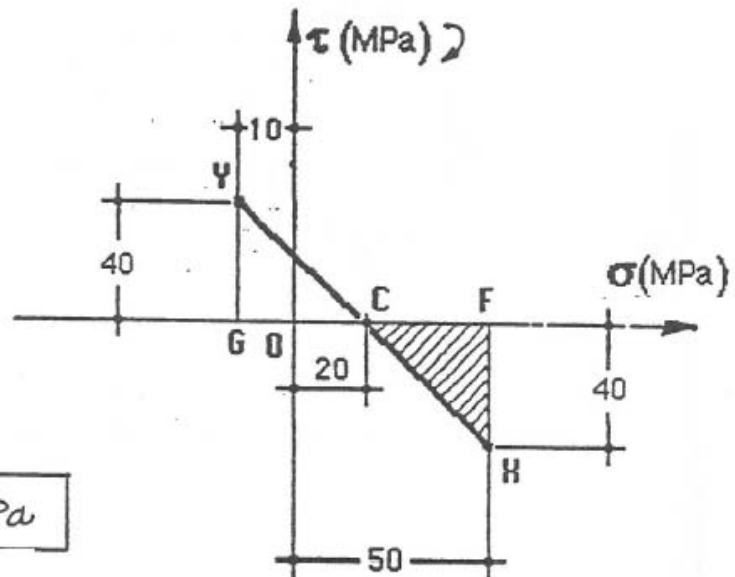
**Solution:**



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20 \text{ MPa}$$

- Locate point X
- Locate Point Y
- From Shaded  $\Delta$
- $CF = 50 - 20 = 30$
- Locate point C
- Radius of the circle :
- $R = \sqrt{(30)^2 + (40)^2} = 50$
- Draw the circle

$$\tau_{max} = R = 50 \text{ MPa}$$

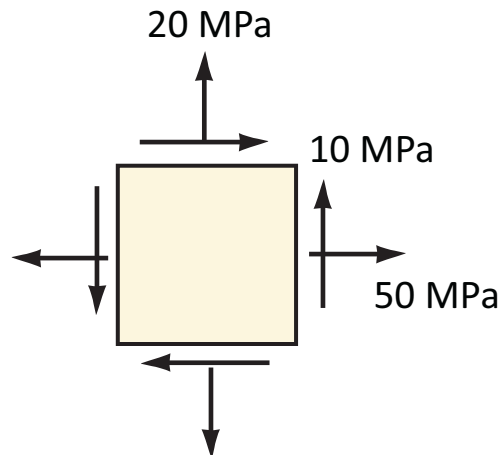




## MECHANICS OF SOLIDS

### PLANE STRESS

#### Problem:



The stresses shown in the figure act at a point in a machine part. Using the given stress element, the maximum inplane shear stress (MPa) in the element is most nearly:

- (A) 16.5
- (B) 18.0
- (C) 21.6
- (D) 28.0

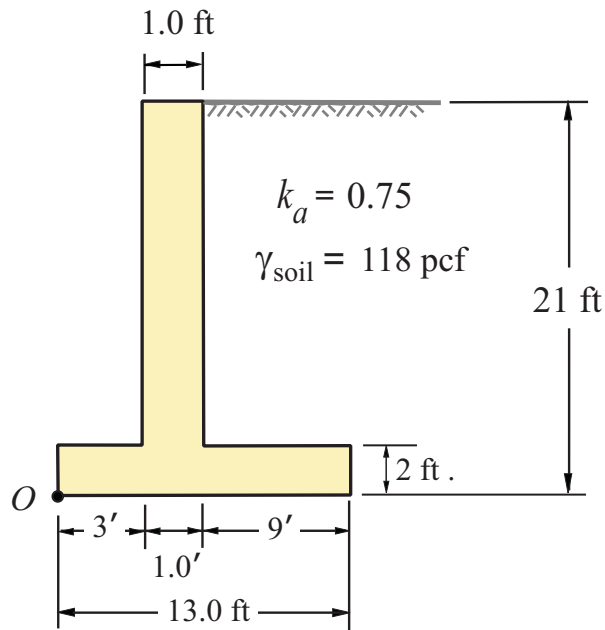
$$\tau_{\max} = ?$$

## CANTILEVER RETAINING WALLS

### FACTOR OF SAFETY (Overturning)

**Problem:** (Retaining Wall)

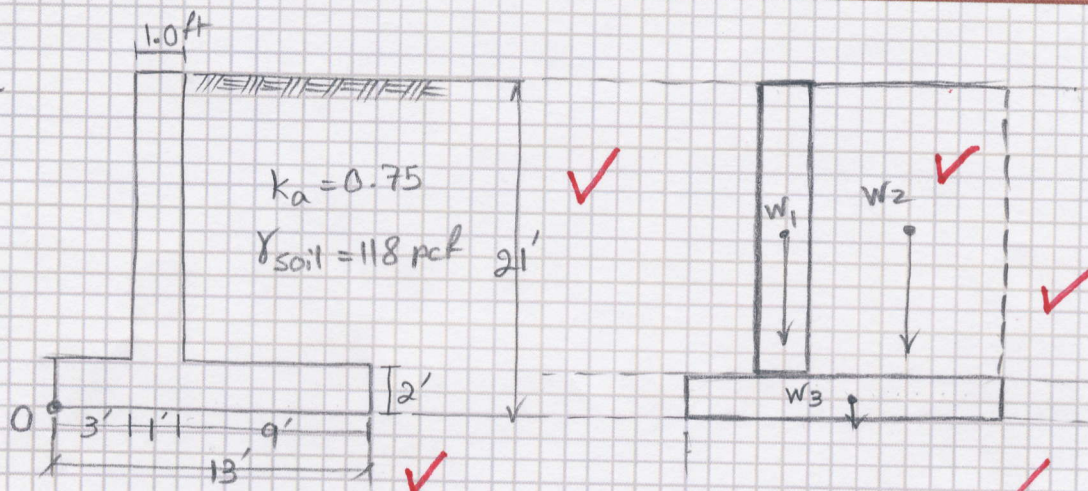
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The dimensions of a cantilever retaining wall are shown in the figure. Using the given Rankine's coefficient  $k_a$  and the other related data as listed determine the following:

- (a) the overturning moment
- (b) the stabilizing moment
- (c) the factor of safety against overturning

Problem 5:



$$H = \frac{k_a \gamma h^2}{2} = \frac{(0.75)(0.118)(21)^2(1)}{2} = \frac{(0.089)(441)(1)}{2} = 19.625 \text{ k}$$

$$\bar{y} = \frac{h}{3} = \frac{21}{3} = 7'$$

$$\gamma = 118 \text{ pcf} = 0.118 \text{ ksf}$$

$$W_1 = (1)(19)(1)(0.150) = 2.850 \text{ kips}$$

$$W_2 = (9)(19)(1)(0.118) = 20.178 \text{ kips}$$

$$W_3 = (2)(13)(1)(0.150) = 3.900 \text{ kips}$$

$$X_1 = 3.5'$$

$$X_2 = 8.5'$$

$$X_3 = 6.5'$$

$$M_{OT} = H \cdot \bar{y} = (19.625)(7) = 137.375 \text{ kip/ft}$$

$$M_s = W_1 X_1 + W_2 X_2 + W_3 X_3$$

$$(2.850)(3.5) + (20.178)(8.5) + (3.900)(6.5) = 9.975 + 171.513 + 25.350 = 206.838 \text{ kip/ft}$$

$$FS = \frac{M_s}{M_{OT}} = \frac{206.838}{137.375} = 1.506$$

**PERFECT  
SOLUTION!**