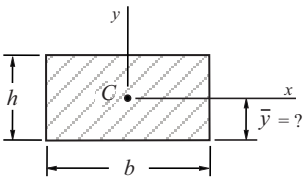
BASIC FORMULAS

Problem: (Rectangle)



C: Centroid
Origin of axes at *C*

(a) Area of the rectangle (A = ?)
(b) Centroid of the rectangle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas:

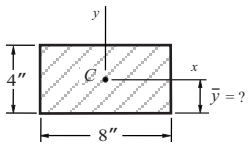
$$A = bh$$

$$\bar{y} = \frac{h}{2}$$

$$I_{cx} = \frac{b\,h^3}{12}$$

$$I_{cy} = \frac{h\,b^3}{12}$$

Example:



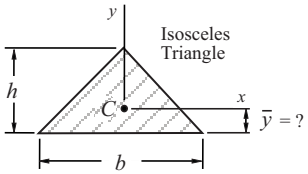
$$A = bh = (8)(4) = 32 \text{ in.}^2$$

$$\bar{y} = \frac{h}{2} = \frac{4}{2} = 2 \text{ in.}$$

$$I_{cx} = \frac{b\,h^3}{12} = \frac{(8)(4)^3}{12} = 42.67 \text{ in.}^4$$

$$I_{cy} = \frac{h\,b^3}{12} = \frac{(4)(8)^3}{12} = 170.67 \text{ in.}^4$$

Problem: (Triangular Area)



C : Centroid
x, *y* : Centroidal axes

(a) Area of the triangle (A = ?)
(b) Centroid of the triangle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas:

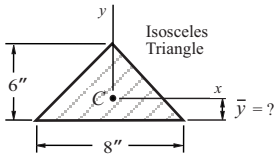
$$A = \frac{bh}{2}$$

$$\bar{y} = \frac{h}{3}$$

$$I_{cx} = \frac{b\,h^3}{36}$$

$$I_{cy} = \frac{h\,b^3}{48}$$

Example:



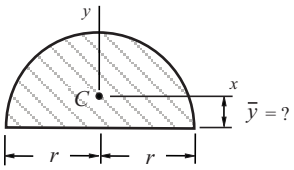
$$A = \frac{bh}{2} = \frac{(8)(6)}{2} = 24 \text{ in.}^2$$

$$\bar{y} = \frac{h}{3} = \frac{6}{3} = 2 \text{ in.}$$

$$I_{cx} = \frac{b\,h^3}{36} = \frac{(8)(6)^3}{36} = 48.00 \text{ in.}^4$$

$$I_{cy} = \frac{h\,b^3}{48} = \frac{(6)(8)^3}{48} = 64.00 \text{ in.}^4$$

Problem: (Half Circle)



(a) Area of the semicircle (A = ?)
(b) Centroid of the semicircle ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Solution:

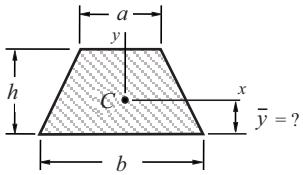
$$A = \frac{\pi r^2}{2} \qquad \bar{y} = \frac{4r}{3\pi}$$

$$I_{cx} = \frac{r^4}{72\pi} \cdot \left(9\,\pi^2 - 64\right)$$

$$I_{cx} \approx 0.109757 \cdot r^4$$

$$I_{cy} = \frac{\pi r^4}{8}$$

Problem: (Trapezoidal Area)



C : Centroid
x, *y* : Centroidal axes

Isosceles
Trapezoid

(a) Area of the trapezoid (A = ?)
(b) Centroid of the trapezoid ($\bar{y}$ = ?)
(c) Moment of inertia about the x axis (I_{cx} = ?)
(d) Moment of inertia about the y axis (I_{cy} = ?)

Formulas

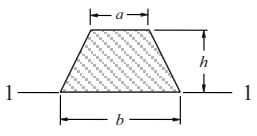
$$A = \frac{h}{2}(a+b) \qquad \bar{y} = \frac{h}{3}\frac{(2a+b)}{(a+b)}$$

Isosceles
Trapezoid

$$I_{cx} = \frac{h^3}{36} \cdot \frac{\left(a^2 + 4ab + b^2\right)}{(a+b)}$$

$$I_{cy} = \frac{h\,(a+b)\,(a^2+b^2)}{48}$$

$$I_{1\text{-}1} = \frac{h^3\,(3a+b)}{12}$$



Isosceles
Trapezoid

Centroid of an Area by Integration

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A y\,dA}{\int_A dA}$$

Moments of Inertia (*I*)

$$I_x = \int_A y^2\,dA \qquad I_y = \int_A x^2\,dA \qquad J_o = I_x + I_y$$

Parallel Axis Theorem (PAT)

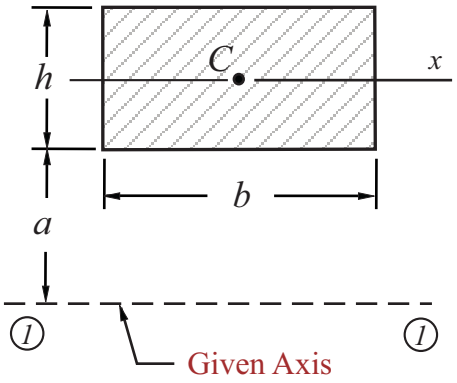
$$I_{cx} = \Sigma I_o + \Sigma A d^2$$
$$I_{cy} = \Sigma I_o + \Sigma A d^2$$

Radius of Gyration (*r*)

$$r_x = \sqrt{\frac{I_x}{A}} \qquad r_y = \sqrt{\frac{I_y}{A}} \qquad r_o = \sqrt{\frac{J_O}{A}}$$

J_o = Polar Moment of Inertia

PARALLEL AXIS THEOREM



PAT | **Parallel Axis Theorem**

(a) Moment of inertia about the x axis ($I_{cx} = ?$)
(b) Moment of inertia about the (1-1) axis ($I_{1\text{-}1} = ?$)

Formulas:

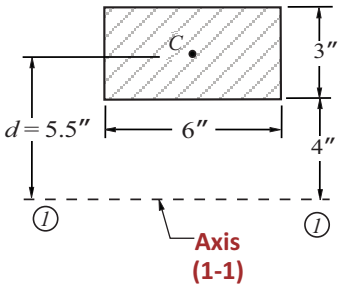
$$I_{cx} = \frac{b\,h^3}{12}$$

$$I_{1\text{-}1} = I_{cx} + A \cdot d^2 \qquad \textbf{PAT}$$

I_{cx} = Moment of inertia about the centroidal x axis
$I_{1\text{-}1}$ = Moment of inertia about axis (1-1)
A = Area ($A = b.h$)
d = Distance from the centroid to the axis (1-1)

$$d = a + h/2$$

Example:



$$I_{cx} = \frac{b\,h^3}{12} = \frac{(6)(3)^3}{12} = 13.50 \text{ in.}^4$$

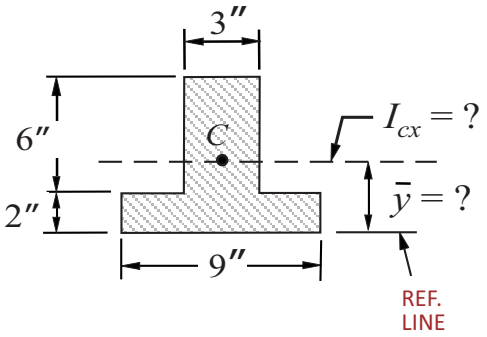
$$A = bh = (6)(3) = 18 \text{ in.}^2$$

$$d = 4 + \tfrac{3}{2} = 5.5 \text{ in.}$$

$$I_{1\text{-}1} = I_{cx} + A.d^2 = 13.50 + (18)(5.5)^2 = 558 \text{ in.}^4$$

$$I_{1\text{-}1} = 558 \text{ in.}^4$$

Problem: (Centroid / Moments of Inertia)



FE/PE EXAMS

The dimensions of a composite area are given as shown in the figure. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 2.30
(B) 3.00
(C) 3.75
(D) 4.15

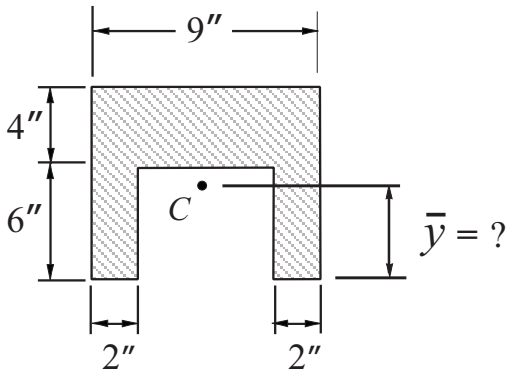
(2) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly (I_{cx})

(A) 142
(B) 204
(C) 257
(D) 346

(3) the moment of inertia (in.^4) about the vertical centroidal axis is most nearly (I_{cy})

(A) 168
(B) 154
(C) 148
(D) 135

Problem: (Centroid / Moments of Inertia)



The dimensions of a composite area are given as shown in the figure. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 7.30
(B) 7.82
(C) 6.75
(D) 6.00

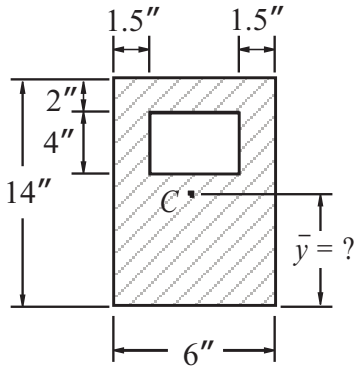
(2) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly (I_{cx})

(A) 642
(B) 504
(C) 480
(D) 395

(3) the moment of inertia (in.^4) about the vertical centroidal axis is most nearly (I_{cy})

(A) 468
(B) 545
(C) 648
(D) 735

Problem: (Centroids / Moments of Inertia)



FE/PE EXAM

The dimensions of a composite area is given as shown. Using the listed data for the solid and the void parts, answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 9.0
(B) 8.5
(C) 7.6
(D) 6.5

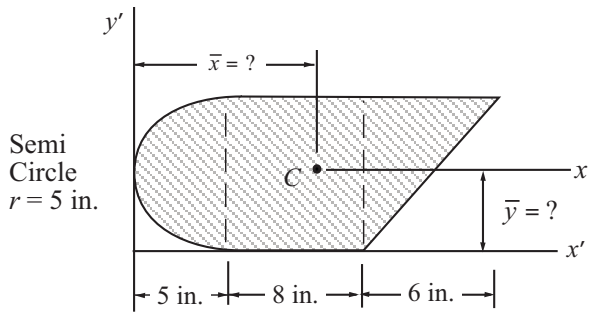
(2) the moment of inertia (in^4) about the horizontal centroidal axis is most nearly

(A) 1050
(B) 1230
(C) 1350
(D) 1400

(3) the moment of inertia (in^4) about the vertical centroidal axis is most nearly

(A) 400
(B) 326
(C) 243
(D) 160

Problem: (Centroid / Moments of Inertia)



FE/PE EXAMS

The dimensions of a composite area are given as shown in the figure. Using the listed data answer the following questions:

(1) the distance $\bar{x}$ (bar) (in.) of the centroid is most nearly

(A) 9.30
(B) 8.60
(C) 7.75
(D) 6.44

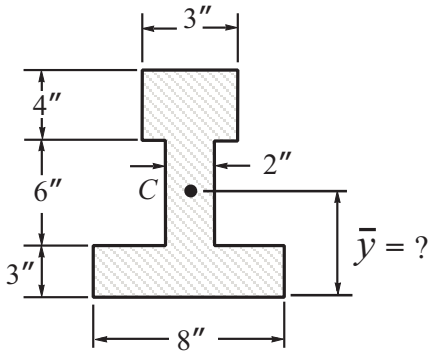
(2) the distance $\bar{y}$ (bar) (in.) of the centroid is most nearly

(A) 7.30
(B) 6.76
(C) 6.14
(D) 5.33

(3) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly (I_{cx})

(A) 1355
(B) 1224
(C) 1145
(D) 1088

Problem: (Centroid / Moments of Inertia)



FE/PE EXAM

The dimensions of a composite area are given as shown in the figure. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 4.65
(B) 5.00
(C) 5.65
(D) 6.15

(2) the moment of inertia ($in.^4$) about the horizontal centroidal axis is most nearly (I_{cx})

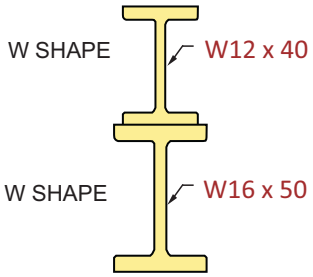
(A) 680
(B) 715
(C) 765
(D) 808

(3) the moment of inertia ($in.^4$) about the vertical centroidal axis is most nearly (I_{cy})

(A) 122
(B) 130
(C) 141
(D) 153

CLASS ASSIGNMENT

CENTROIDS / MOMENTS of INERTIA



FE EXAM

Two W Shapes are welded together to form a solid composite section as shown. Using the listed data and the new NCEES Reference Handbook's W-Shape tables, answer the following questions:

(1) the location (in.) of the centroid from the bottom is most nearly

(A) 15.8
(B) 14.4
(C) 13.9
(D) 12.1

(2) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly

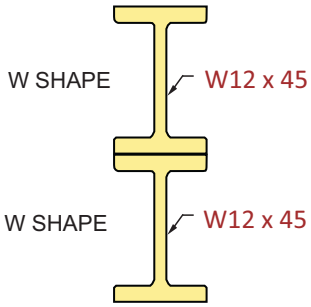
(A) 1920
(B) 2261
(C) 2655
(D) 3020

(3) the moment of inertia (in.^4) about the vertical centroidal axis is most nearly

(A) 68.45
(B) 76.52
(C) 81.30
(D) 90.25

CLASS ASSIGNMENT

CENTROIDS / MOMENTS of INERTIA



FE EXAM

Two W12x45 sections are welded together to form a solid composite section as shown. Using the listed data and the new NCEES Reference Handbook's W-Shape tables, answer the following questions:

(1) the location (in.) of the centroid from the bottom is most nearly

(A) 10.8
(B) 14.1
(C) 13.0
(D) 12.1

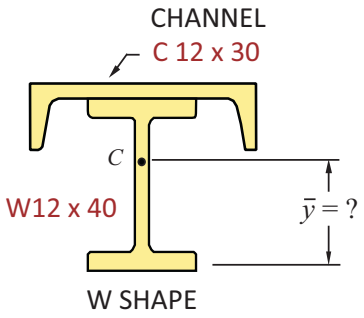
(2) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly

(A) 1120
(B) 1228
(C) 1655
(D) 1820

(3) the moment of inertia (in.^4) about the vertical centroidal axis is most nearly

(A) 180
(B) 160
(C) 120
(D) 100

Problem: (Centroid / Moments of Inertia)



FE/PE EXAM

A W-shape section and a channel are welded together to form a composite section as shown. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 9.12
(B) 8.43
(C) 7.64
(D) 6.93

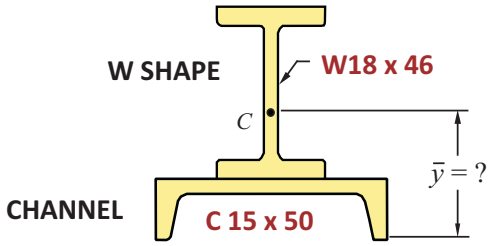
(2) the moment of inertia (in^4) about the horizontal centroidal axis is most nearly

(A) 588.7
(B) 530.3
(C) 480.6
(D) 425.2

(3) the moment of inertia (in^4) about the vertical centroidal axis is most nearly

(A) 185.5
(B) 206.1
(C) 244.8
(D) 320.0

Problem: (Centroid/Moments of Inertia)



FE/PE EXAM

A W-shape section and a channel are welded together to form a composite section as shown. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 9.0
(B) 8.5
(C) 7.6
(D) 6.9

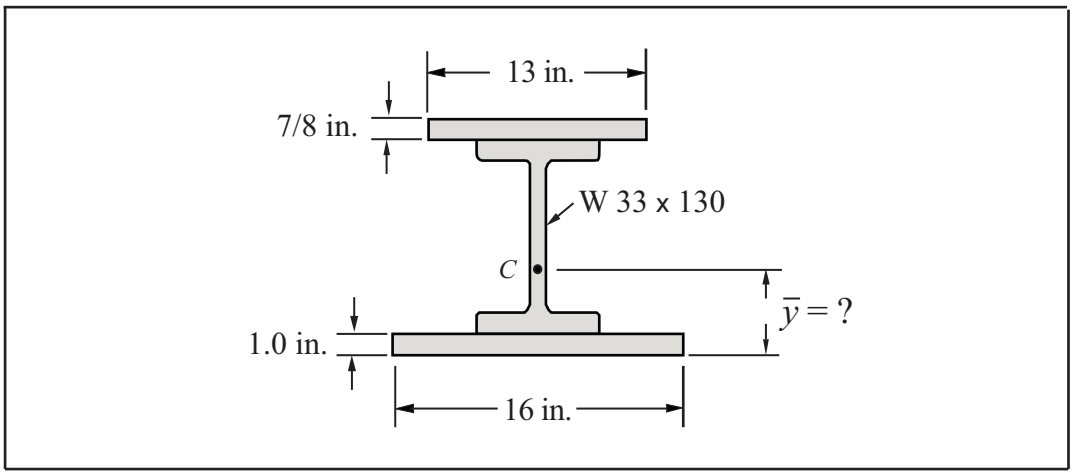
(2) the moment of inertia (in^4) about the horizontal centroidal axis is most nearly

(A) 1250
(B) 1300
(C) 1350
(D) 1400

(3) the moment of inertia (in^4) about the vertical centroidal axis is most nearly

(A) 385
(B) 426
(C) 524
(D) 560

Centroid / Moment of Inertia



Centroid Calculations

	A_i	y_i	$A_i y_i$
	in.^{2}	in.	in.^{3}
1	11.375	34.527	392.745
2	38.300	17.545	671.973
3	16.000	0.500	8.000
Σ	65.675	—	1072.718

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{1072.718}{65.675} = 16.33 \text{ in.}$$

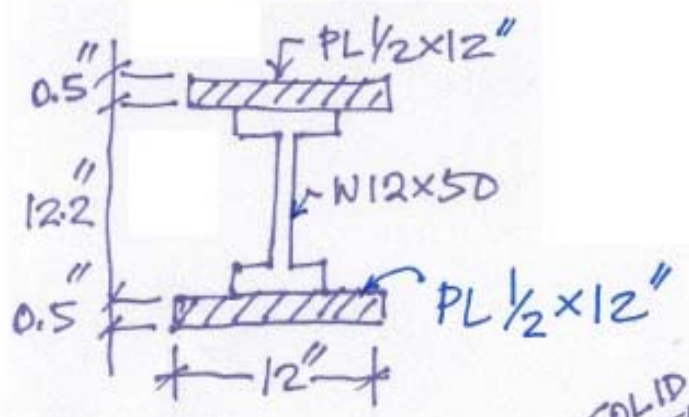
Moment of Inertia

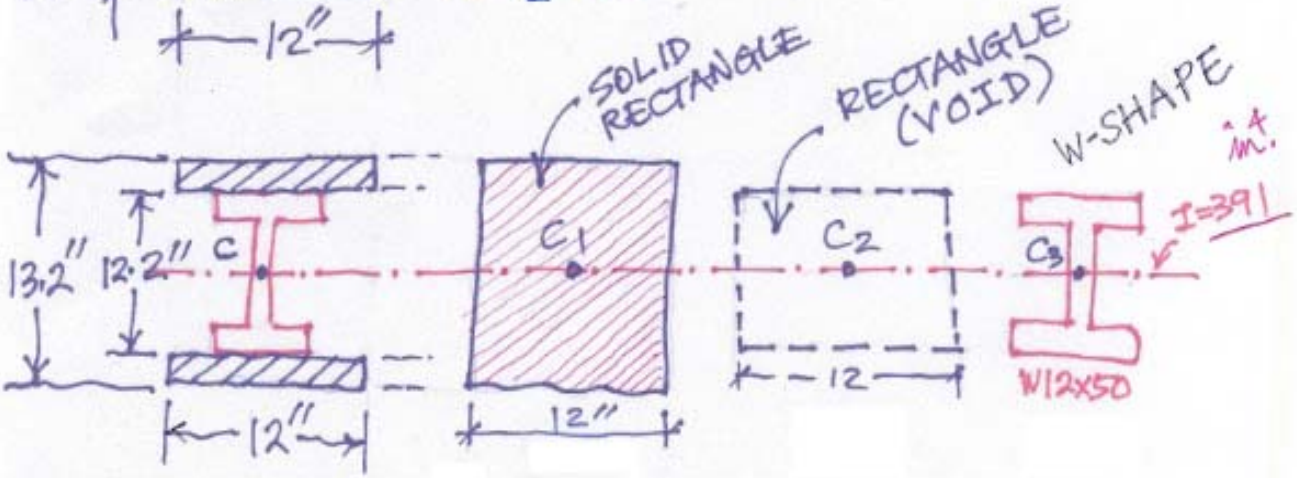
	I_o	A_i	d_i	$A_i d_i^2$
	in.^{4}	in.^{2}	in.	in.^{4}
1	0.829	11.375	18.194	3765
2	6710	38.300	1.211	56.17
3	1.333	16.000	15.834	4011
Σ	6,712	65.675	—	7,833

$$I_x = I_o + A_i \cdot d_i^2 = 6{,}712 + 7{,}833 = 14{,}545 \text{ in.}^4$$

MOMENT of INERTIA

FASTEST METHOD





(I) (II) (III)

CASE-I : $I_1 = bh^3/12 = (12)(13.2)^3/12 = 2300\ in^4$

CASE-II : $I_2 = -bh^3/12 = (12)(12.2)^3/12 = -1816\ in^4$

CASE-III : $I_3 = 391\ in^4$ (from Table)

$$I = I_1 - I_2 + I_3$$

$$= 2300 - 1816 + 391$$

$$\boxed{I_{cx} = 875\ in^4}$$

ANSWERS

Dr. Z's CORNER / October 2014

PDF-PROBLEMS & EXAMPLES

Answers to selected problems:

Sheet #	(1)	(2)	(3)
DEF-69	*B*	*D*	*B*
FRCT-135	*D*	*C*	*B*
FRCT-132	*C*	*D*	*A*