ASCE NATIONAL CAPITAL SECTION eNewsletter
National Capital Section
FOUNDED 1916

APRIL 2015

DR. Z's CORNER

Conquering the FE & PE exams
Formulas, Examples & Applications

Topics covered in this month's column:

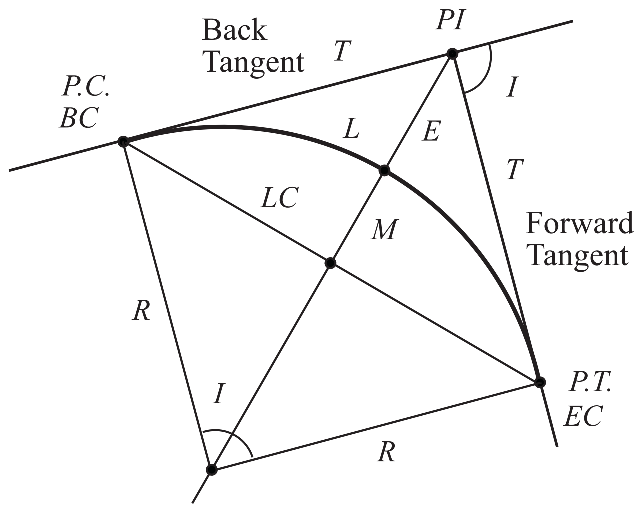
- PE Exam Specifications (Geotechnical)
- Transportation (Horizontal Curves)
- Structural Design (LRFD, Steel Design, Columns, Compression Members and Tension Members)
- Structural Analysis (LRFD, Factored Loads & Moments)
- Structural Design (LRFD, Reinforced Concrete Beams)
- Statics and Mechanics of Materials
- Math (First Order Ordinary Differential Equations)
- Technology Usage (Calculator, Casio-115-ES-PLUS)
- Regression Analysis (Calculator, Casio-115-ES-PLUS)
- Converting Binary and Decimal Numbers
- Triangular Area Computations Using Determinants

PE Civil Geotechnical
Afternoon Depth Exam Specifications

Beginning and effective April 2015 examinations

- **Site Characterization:** 5 questions,
- **Soil Mechanics, Laboratory Testing, and Analysis:** 5 questions,
- **Field Materials Testing, Methods, and Safety:** 3 questions,
- **Earthquake Engineering and Dynamic Loads:** 2 questions,
- **Earth Structures:** 4 questions,
- **Groundwater and Seepage:** 3 questions,
- **Problematic Soil and Rock Conditions:** 3 questions,
- **Earth Retaining Structures (ASD or LRFD):** 5 questions,
- **Shallow Foundations (ASD or LRFD):** 5 questions,
- **Deep Foundations (ASD or LRFD):** 5 questions,

TYPES OF HORIZONTAL CURVES



NCEES-REF HANDBOOK PAGE-166

Degree of Curve is not shown

I = Intersection angle (also called Δ)
D = Degree of Curve (Arc Definition)
PI = Point of Intersection
PC = Point of Curvature, also called (*BC*)
PT = Point of Tangency, also called (*EC*)
L = Length of Curve (from *PC* to *PT*)
T = Tangent Distance
E = External Distance
R = Radius
LC = Length of Chord
M = Length of Middle Ordinate

(1) Simple Curve

The simple curve is an arc of circle. The radius (R) determines the sharpness or flatness of the curve.

(2) Compound Curve

The compund curve consists of two simple curves joined together curving in the same direction.

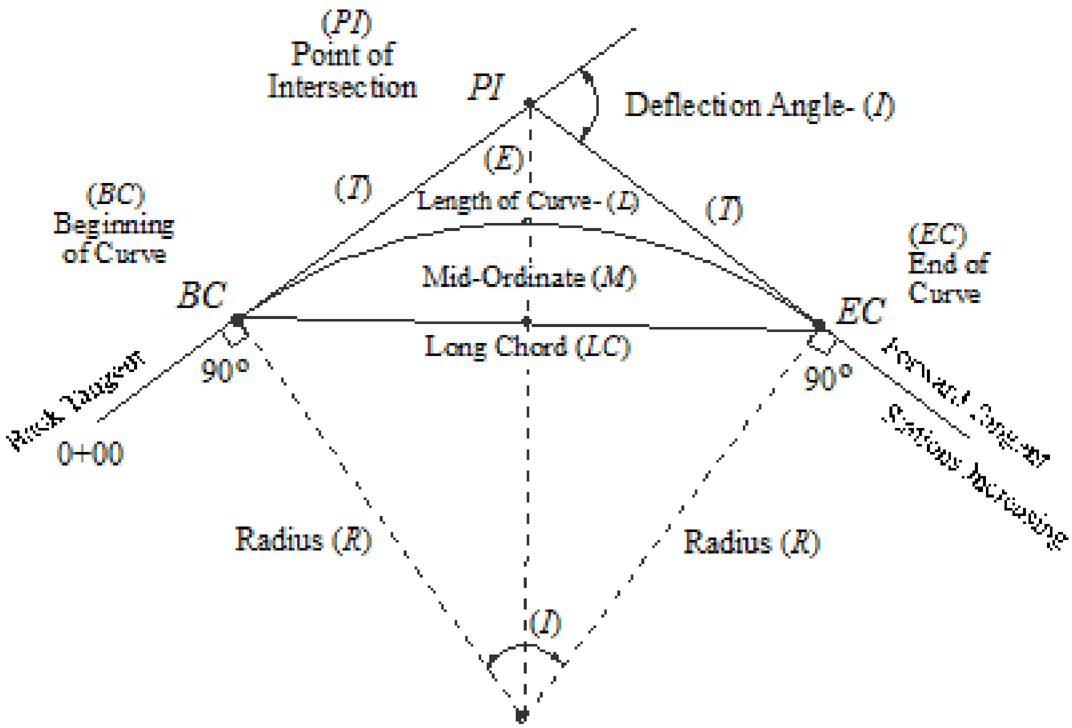
(3) Reverse Curve

A reverse curve consists of two simple curves joined together, but curving in opposite direction. For safety reasons the use of this curve should be avoided.

(4) Spiral Curve

The spiral curve is a curve that has a varying radius. It is used on most modern highways and railroads. It is used for transition from the tangent to a simple curve or between simple curves.

HORIZONTAL (CIRCULAR) CURVES
IMPORTANT FORMULAS



$$R = \frac{5729.58}{D}$$

$$R = \frac{LC}{2\sin(I/2)}$$

$$T = R\tan(I/2)$$

$$T = \frac{LC}{2\cos(I/2)}$$

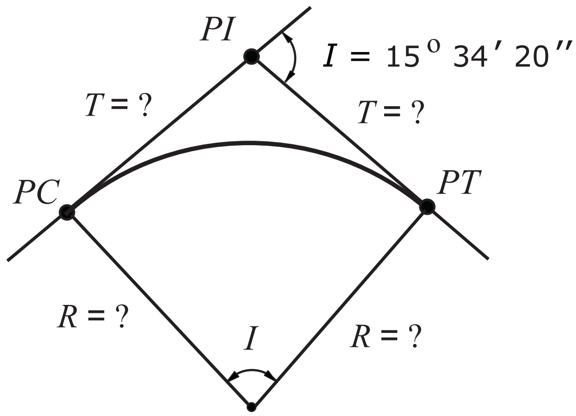
$$L = RI\frac{\pi}{180} = \frac{I}{D}100$$

$$M = R[1 - \cos(I/2)]$$

I = Intersection angle (also called Δ)
D = Degree of Curve (Arc Definition)
PI = Point of Intersection
PC = Point of Curvature, also called (BC)
PT = Point of Tangency, also called (EC)
L = Length of Curve (from PC to PT)
T = Tangent Distance
E = External Distance
R = Radius
LC = Length of Chord
M = Length of Middle Ordinate

TRANSPORTATION

HORIZONTAL CURVE DESIGN



D = 3° 30′ 00″
I = 15° 34′ 20″
sta PI = 54+68.32

A 3° 30′ horizontal curve has forward and backward tangents that intersect at station 54 + 68.32. Knowing that the intersection angle is I = 15° 34′ 20″, determine the following:

(a) the radius of the curve (R)
(b) the tangent distance (T)
(c) the length of long chord (LC)
(d) the external distance (E)
(e) the length of middle ordinate (M)
(f) the length of curve (L)
(g) the station of PC, sta (PC)
(h) the station of PT, sta (PT)

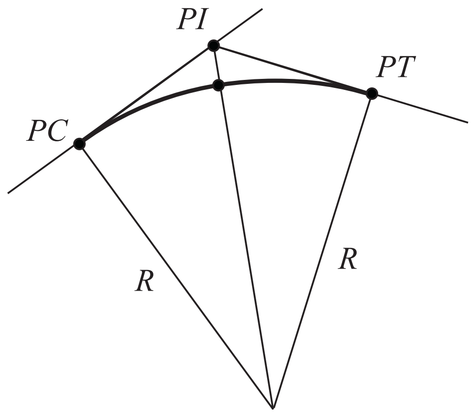
NCEES REFERENCE HANDBOOK PAGE-166

Answers:

a) the radius of curve : $R = 1637.02$ ft.
(b) the tangent distance : $T = 223.84$ ft.
(c) the length of long chord : $LC = 443.55$ ft.
(d) the external distance : $E = 15.23$ ft.
(e) the length of middle ordinate : $M = 15.09$ ft.
(f) the length of curve : $L = 444.92$ ft.
(g) the station of PC : sta (PC) : 52 + 44.48
(h) the station of PT : sta (PT) : 56 + 89.40

TRANSPORTATION

HORIZONTAL CURVES



Sta *PI* = 16+88.35

$I = 12^{o}\ 24'\ 36''$
$D = 5^{o}$

I = Intersection Angle

A 5^{o} horizontal curve has forward and back tangents that intersect at station 16+88.35. Knowing that the intersection angle is $I = 12^{o}\ 24'\ 36''$, answer the following questions:

(1) the radius (ft) of the curve is most nearly, *R*

- (A) 1975.60
- (B) 1694.42
- (C) 1491.53
- (D) 1145.92

(2) the tangent distance (ft) of the curve is most nearly, *T*

- (A) 247.62
- (B) 137.54
- (C) 124.59
- (D) 101.18

(3) the station of point of curve - *PC* is most nearly, *PC*

- (A) 10 + 94.65
- (B) 12 + 85.44
- (C) 15 + 63.76
- (D) 17 + 10.15

(4) the station of point of tangent - *PT* is most nearly, *PT*

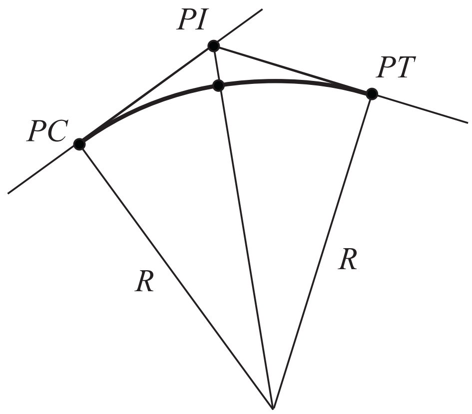
- (A) 13 + 94.24
- (B) 15 + 88.16
- (C) 16 + 77.82
- (D) 18 + 11.96

Important note for students:

This problem is for practice and has four questions. In real FE & PE exams, each problem will have ONLY ONE question.

TRANSPORTATION

HORIZONTAL CURVES



Sta *PI* = 18+26.44

I = 26° 24′
E = 62.2 ft

I = Intersection Angle

Two segments of a transportation route intersect at station 18+26.44 with a deflection angle of 26° 24′. If the external distance *E* from the *PI* to the centerline of the highway is measured as 62.2 ft, determine the following:

(1) the radius (ft) of the curve is most nearly, R

(A) 1675.60
(B) 1994.45
(C) 2291.83
(D) 2536.71

(2) the degree of the curve (deg-min) is most nearly, D

(A) 4 deg 40 min
(B) 3 deg 30 min
(C) 3 deg 18 min
(D) 2 deg 30 min

(3) the tangent distance (ft) of the curve is most nearly, T

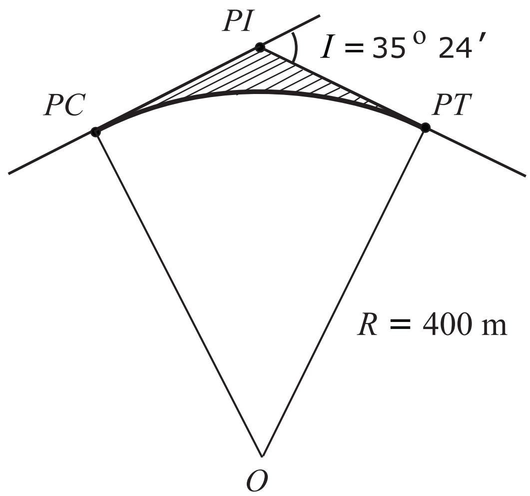
(A) 447.62
(B) 537.54
(C) 588.26
(D) 651.18

(4) the station of point of curvature-*PC* is most nearly, *PC*

(A) 10 + 94.90
(B) 12 + 88.90
(C) 14 + 77.82
(D) 18 + 10.47

TRANSPORTATION

HORIZONTAL CURVES



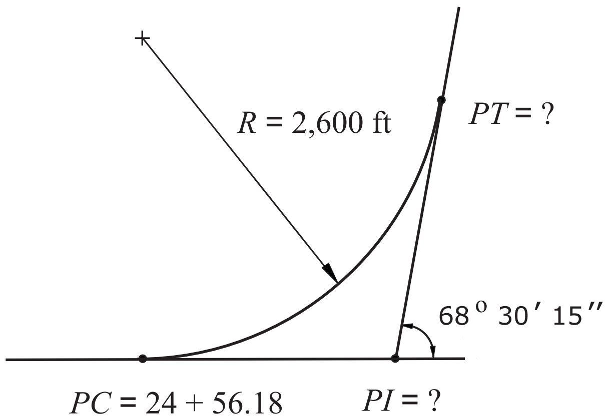
R = 400 m
I = 35° 24′

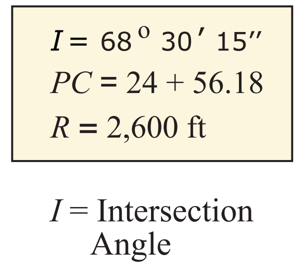
I = Intersection Angle

A horizontal curve is given as shown above. The intersection angle and the radius of the curve are given as listed. The shaded area (m^2) between the circular curve and the tangents is most nearly:

(A) 4,879
(B) 3,577
(C) 2,427
(D) 1,635

Problem: (Transportation / Horizontal Curves)





Referring to the horizontal curve shown in the figure above, answer the following questions:

(1) the tangent distance (ft) of the curve is most nearly, T

(A) 1575.20
(B) 1770.42
(C) 1886.36
(D) 2026.14

(2) the external distance (ft) of the curve is most nearly, E

(A) 545.53
(B) 670.42
(C) 786.36
(D) 826.14

(3) the station of point of intersection-*PI* is most nearly, PI

(A) 32 + 34.55
(B) 38 + 42.24
(C) 42 + 26.60
(D) 45 + 18.47

(4) the station of point of tangency-*PT* is most nearly, PT

(A) 39 + 26.45
(B) 43 + 31.16
(C) 45 + 16.55
(D) 55 + 64.81

Important note for students:

This problem is for practice and has four questions. In real FE & PE exams, every problem will have ONLY ONE question.

COMPRESSION MEMBERS
COLUMN FORMULAS

$$P_u \le \phi_c P_n$$

$$P_u \le \phi_c P_n \le \phi_c A_g F_{cr}$$

P_u = Total Factored Loads

P_n = *Nominal C*ompressive Strength

$\phi_c P_n$ = *Design* Compressive Strength

F_{cr} = Critical Buckling Stress

A_g = Gross Area of the Cross-Section

ϕ_c = Resistance Factor (for columns $\phi_c = 0.90$)

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$

F_e = Euler's Stress (elastic buckling stress)

K = Effective Length Factor

L = Column Length

KL = Effective Column Length

r = Radius of Gyration (always min. value)

IMPORTANT STEEL COLUMN FORMULAS

$$P_u \le \phi_c P_n$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$\phi_c = 0.90$

Resistance Factor

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$

$$\frac{KL}{r} \le 4.71 \sqrt{\frac{E}{F_y}} \qquad F_{cr} = 0.658^{\left(\frac{F_y}{F_e}\right)} F_y$$

$$\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}} \qquad F_{cr} = 0.877\, F_e$$

K = Effective Length Factor (AISC)

L = Column Length

KL = Effective Column Length

KL/r_{min} = slenderness ratio

r_{min} = minimum radius of gyration

F_y = Yield Stress of Steel (ksi)

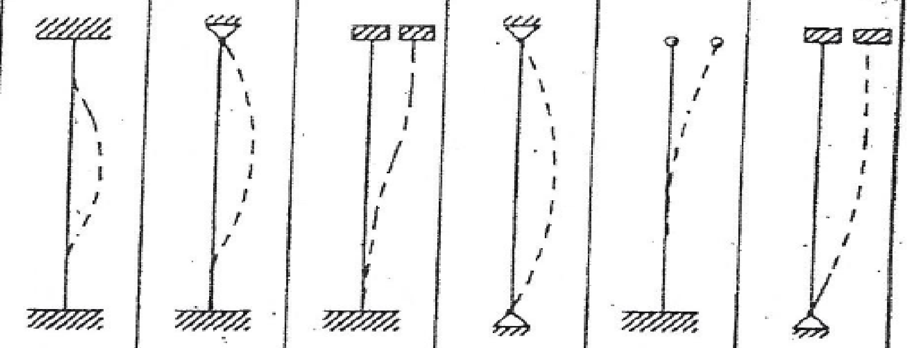
F_e = Euler stress or Elastic Buckling Stress (ksi)

F_{cr} = Critical Stress (ksi)

E = Modulus of Elasticity of Steel

AISC TABLES / CHARTS FOR COMPRESSION MEMBERS

EFFECTIVE LENGTH FACTORS (*K*)

BUCKLED SHAPE OF COLUMN SHOWN BY DASHED LINE						
THEORETICAL OR IDEAL "K"	.5	.7	1.0	1.0	2.0	2.0
RECOMMENDED "K"	.65	.80	1.2	1.0	2.1	2.0

END CONDITION CODE	
(hatched)	FIXED - BOTH ROTATION & TRANSLATION
(pin)	PINNED - ROTATION FREE BUT TRANSLATION FIXED
(box)	ROTATION FIXED BUT TRANSLATION FREE
o	FREE - ROTATION AND TRANSLATION

NCEES Reference Handbook, page-158 (Adapted from AISC, Steel manual, 2011)

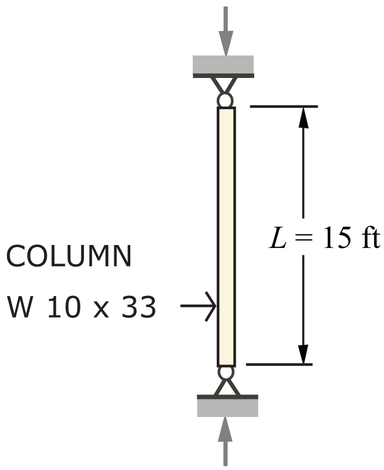
$$\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{F_y}} \qquad F_{cr} = 0.658^{\left(\frac{F_y}{F_e}\right)} F_y$$

AISC E 3-2

$$\frac{KL}{r} > 4.71\sqrt{\frac{E}{F_y}} \qquad F_{cr} = 0.877\ F_e$$

AISC E 3-3

Problem: (Design Strength)



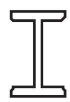
W 10 x 33
ASTM-A992 Steel
Both Ends Pinned

$E = 29 \times 10^3$ ksi

(a) Find the slenderness ratio (KL/r)
(b) Find the Euler Stress (F_e)
(c) Find the critical buckling stress (F_{cr})
(d) Find the nominal compressive strength (P_n)
(e) Find the design compressive strength ($\phi_c P_n$)

Solution:

W 10 x 33



Steel / A992
$F_y = 50$ ksi
$F_u = 65$ ksi

$A_g = 9.71$ in^2
$r_x = 4.19$ in.
$r_y = 1.94$ in.

$r_{min} = 1.94$ in.

The Slenderness Ratio: (KL / r) Here K = 1.0, NCEES-Ref. Book / Page 158

KL / r_{min} = 1.0 x (15 x 12) / 1.94 = 92.78 < 200   O.K.

The Criteria for the Critical Buckling Stress Equation: (F_{cr})

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113.4$$

$(KL/r_{min}) = 92.78 < 113.4$, Use AISC Equation E 3.2

The Euler Stress (F_e)

$$F_e = \frac{\pi^2 E}{(KL/r_{min})^2} = \frac{\pi^2 (29{,}000)}{(92.78)^2} = 33.25 \text{ ksi}$$

STRESS RATIO:

$F_y / F_e = 50 / 33.25 = 1.504$

The Critical Buckling Stress: (F_{cr})

$F_{cr} = (0.658^{F_y/F_e}) \cdot F_y = (0.658)^{(1.504)}\ (50) = 26.65$ ksi   (AISC Equation E 3.2)

The Nominal Compressive Strength: (P_n)

$P_n = F_{cr} \cdot A_g\ F_{cr} = (26.65)\ (9.71) = 258.77$ kips

The Design Compressive Strength: ($\phi_c P_n$)

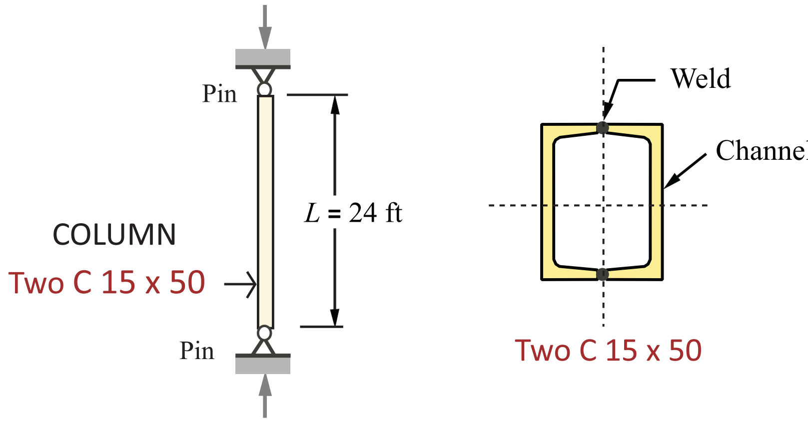
$\phi_c P_n = 0.90 \times 258.77 = 232.9$ kips

$\phi_c P_n = 232.9$ kips

DESIGN OF STEEL STRUCTURES

COMPRESSION MEMBERS

Problem: Steel Column (W)



Two C-Channels are welded together at flanges

Two C 15 x 50 channels are welded together to form a column as shown in the figure. Using the data for the cross section and the column length, answer the following questions:

(1) The min. area moment of inertia (in^4) is most nearly:

(A) 211.20
(B) 272.85
(C) 312.53
(D) 355.10

$I_{min} = ?$

(2) The minimum radius of gyration (in.) is most nearly:

(A) 4.60
(B) 4.08
(C) 3.60
(D) 3.05

$r_{min} = ?$

(3) The slenderness ratio of the column is most nearly:

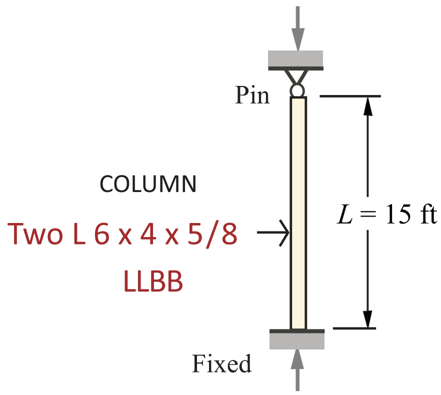
(A) 98.50
(B) 94.43
(C) 85.24
(D) 82.42

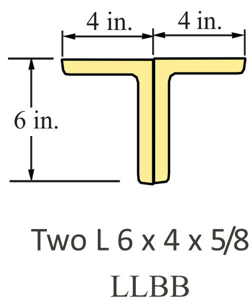
$KL/r_{min} = ?$

DESIGN OF STEEL STRUCTURES

COMPRESSION MEMBERS

Problem: Steel Column (W)





LLBB = Long Legs Back to Back

Two L 6 x 4 x 5 / 8 angles are welded together to form a column as shown in the figure. Using the data for the cross section and the column length, answer the following questions:

(1) The min. area moment of inertia (in^4) is most nearly:

(A) 20.15
(B) 27.40
(C) 34.57
(D) 46.10

$$I_{\min} = ?$$

(2) The minimum radius of gyration (in.) is most nearly:

(A) 4.20
(B) 3.18
(C) 2.45
(D) 1.53

$$r_{\min} = ?$$

(3) The slenderness ratio of the column is most nearly:

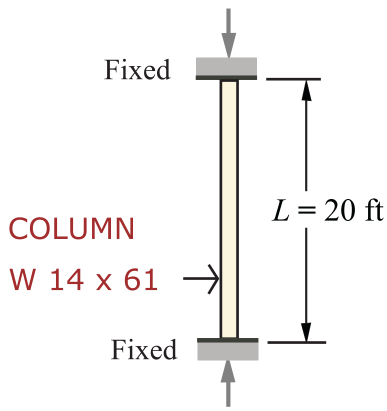
(A) 98.55
(B) 94.12
(C) 86.10
(D) 81.35

$$KL/r_{\min} = ?$$

DESIGN OF STEEL STRUCTURES

COMPRESSION MEMBERS

Problem: Steel Column (W)



W 14 x 61
ASTM-A992
E = 29 x 10^3 ksi
Top end fixed
Bottom end fixed

The W 14 x 61 steel column is 20 ft. long and supported as shown. Using the listed data answer the following questions:

(1) The slenderness ratio of the column is most nearly:

(A) 46.57
(B) 58.36
(C) 63.67
(D) 76.40

$$KL/r_{min} = ?$$

(2) The design strength of the column (kips) is most nearly:

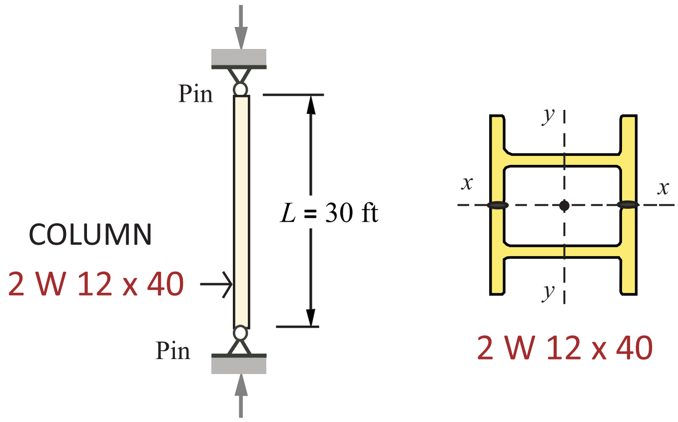
(A) 686
(B) 599
(C) 495
(D) 425

$$\phi P_n = ?$$

DESIGN OF STEEL STRUCTURES

COMPRESSION MEMBERS

Problem: Steel Column (W)



Two W-shapes are welded together at flanges

Two W 12 x 40 shapes are welded together to form a column as shown in the figure. Using the data for the cross section and the column length, answer the following questions:

(1) The min. area moment of inertia (in^4) is most nearly:

(A) 515.22
(B) 463.54
(C) 344.57
(D) 286.10

$I_{\min} = ?$

(2) The minimum radius of gyration (in.) is most nearly:

(A) 3.20
(B) 3.88
(C) 4.45
(D) 4.86

$r_{\min} = ?$

(3) The slenderness ratio of the column is most nearly:

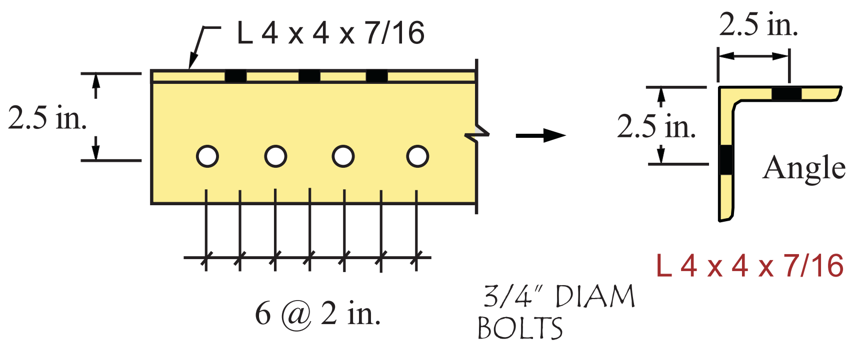
(A) 96.55
(B) 92.48
(C) 88.15
(D) 80.90

$KL/r_{\min} = ?$

STEEL DESIGN / LRFD

TENSION MEMBERS / DESIGN STRENGTH

Problem:



STEEL
A36

A L 4 x 4x 7/16 tension member is connected with 3/4-in. diam. bolts as shown in the figure. If A36 steel is used and both legs of the angle are connected, answer the following questions:

(1) The effective net area (in.^{2}) of the member is most nearly

(A) 2.15
(B) 2.30
(C) 2.64
(D) 3.85

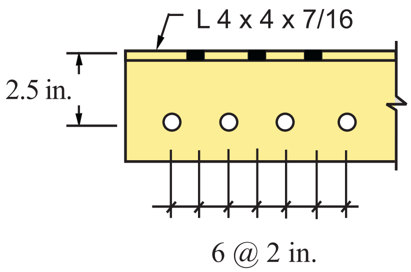
$A_e = ?$

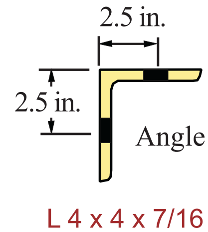
(2) The tensile design strength (kips) of the member is most nearly

(A) 107.0
(B) 110.5
(C) 125.4
(D) 140.0

$\phi P_n = ?$

Problem:





Bolts
3/4 in. diam.
A36 Steel

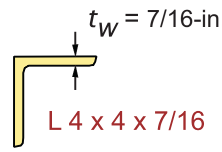
A L 4 x 4x 7/16 tension member is connected with 3/4-in. diam. bolts as shown in the figure. If A36 steel is used and both legs of the angle are connected, determine the design strength.

Solution:

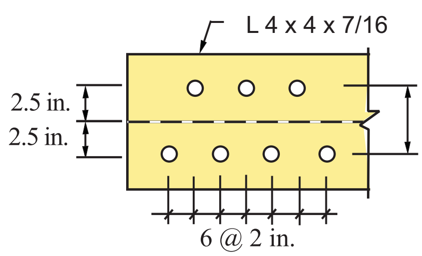
Angle

L 4 x 4 x 7/16

L 4 x 4 x 7/16
$A = A_{gross} = 3.31$ in^2. (from manual)
A36 $\rightarrow$ $F_y = 36$ ksi, $F_u = 58$ ksi



Unfold both legs of the angle and find the gage distance (g)



ANGLE $\rightarrow$ PLATE

g = Gage Distance

$g = 2.5 + 2.5 - 7/16 = 4.563$ in.

Hole diameter:

$d_{hole} = d_{bolt} + 1/8 = 3/4 + 1/8 = 7/8$-in.

Effective net area: (A_e)

$A_{net} = A_{gross} - d.t - d'.t = 3.31 - 7/8\ (7/16) - 7/16\ [\ 7/8 - 2^2/(4x4.563)\ ]\ = 2.64$ in.^{2}

Since the both legs of the angle are connected: $A_n = A_e$

$A_{eff} = A_{net} = 2.64$ in.^{2}

The design strength based on yielding or gross area: ($\phi_t\, P_n$)

$\phi_t\, P_n = 0.90\, F_y\, A_{gross} = 0.90\,(36)\,(3.31) = 107.2$ kips = 107 kips $\leftarrow$

The design strength based on fracture or effective area: ($\phi_t\, P_n$)

$\phi_t\, P_n = 0.75\, F_u\, A_{eff.} = 0.75\,(58)\,(2.64) = 114.8$ kips

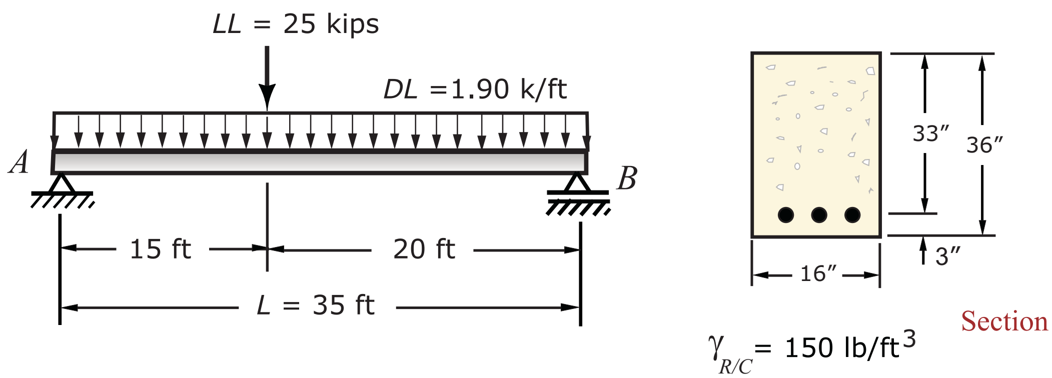
The design strength is the smaller value.

$\boxed{\phi_t\, P_n = 107 \text{ kips}}$

STRUCTURES / REINFORCED CONCRETE

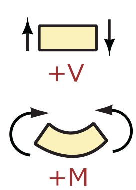
FACTORED LOADS / MAX. FACTORED MOMENT

Service Loads



The beam weight will be included in the computations:

A simply supported R/C beam is loaded as shown. Considering the weight of the concrete beam, the maximum FACTORED bending moment (k-ft) is most nearly:



(A) 520
(B) 593
(C) 686
(D) 793

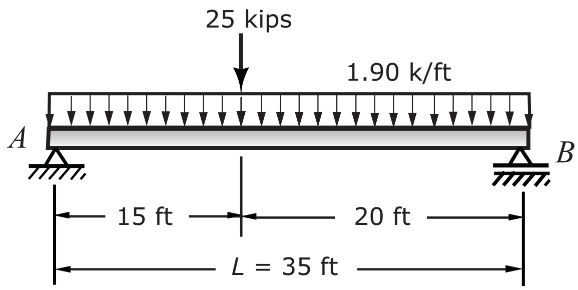
$M_{u_{max}} = ?$

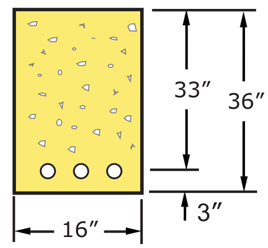
COMPLETE SOLUTION

MCBM-325
ZEYTINCI
SPRING 2015

FACTORED LOADS / FACTORED MOMENTS

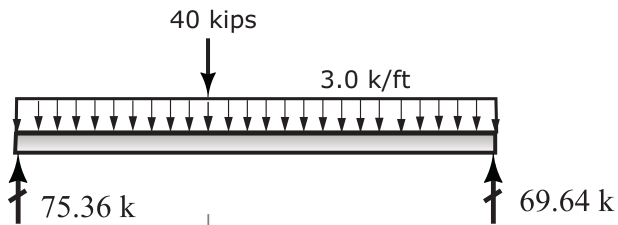
Service Loads





Section

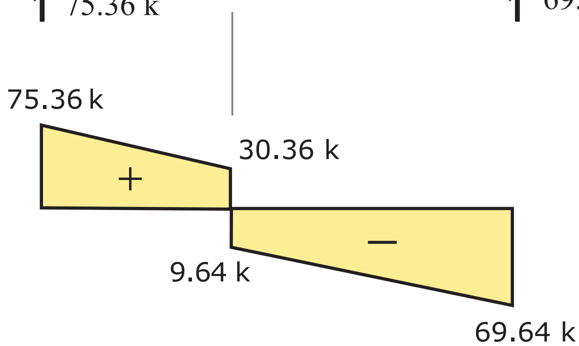
Factored Loads



Beam Weight

$BM = 600$ lb/ft

Shear Diagram



(V)

Beam Weight

$$BM = \frac{(16)(36)}{144}(150) = 600 \text{ lb/ft} = 0.6$$

Factored Dead Load

$$w_u = 1.2\,(0.6 + 1.9) = 1.2\,(2.5) = 3.0 \text{ k/ft}$$

Factored Live Load

$$w_u = 1.6\,(25 \text{ kips}) = 40 \text{ kips}$$

Maximum Factored Moment

$$M_u = 0.5\,(15)\,(75.36 + 30.36) = 792.9 \text{ ft-kips}$$

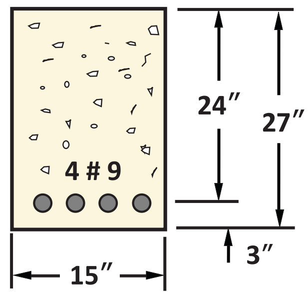
$M_u = 792.9$ ft-kips

DESIGN OF CONCRETE STRUCTURES

LRFD-DESIGN MOMENT CAPACITY

Problem:

FE/PE EXAM



f'_c = 4,000 psi
f_y = 60,000 psi
γ_c = 150 lb/ft^3

The dimensions of a R/C beam section is given as shown. Using the listed data answer the following:

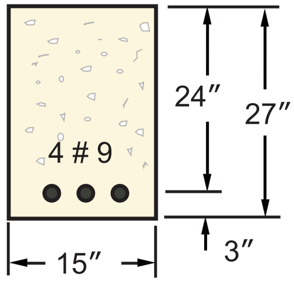
The design moment capacity ϕM_n is most nearly

(A) 220 ft-kips
(B) 285 ft-kips
(C) 320 ft-kips
(D) 390 ft-kips

COMPLETE SOLUTION

REINFORCED CONCRETE BEAMS

LRFD-DESIGN MOMENT CAPACITY



f'_c = 4,000 psi
f_y = 60,000 psi

Determine the design moment capacity ϕM_n of the R/C beam shown in the figure. Concrete and steel properties are as listed.

Solution:

The steel percentage (ρ)

$$\rho = \frac{A_s}{bd} = \frac{4.00}{(15)(24)} = 0.0111 \rightarrow \left| \begin{array}{l} \rho = 0.0111 \\ f'_c = 4{,}000 \text{ psi} \\ f_y = 60{,}000 \text{ psi} \end{array} \right\} \left| \begin{array}{l} \text{From Table} \\ \rho_{min} = 0.0033 \\ \rho_{max} = 0.0181 \end{array} \right\}$$

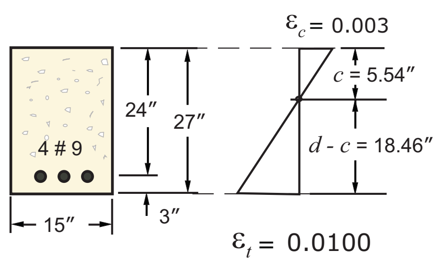
0.0033 < 0.0111 < 0.0181 OK

The depth of the stress block:

$$a = \frac{A_s f_y}{0.85 f'_c\, b} = \frac{(4.00)(60{,}000)}{(0.85)(4000)(15)} = 4.71 \text{ in.}$$

$\beta_1 = 0.85$ since $f'_c = 4000$ psi

$$c = \frac{a}{\beta_1} = \frac{4.71}{0.85} = 5.54 \text{ in.}$$



Drawing the Strain Diagram

$$\varepsilon_t = \frac{d - c}{c}(0.003) = \frac{(24 - 5.54)}{5.54}(0.003) = 0.0100$$

$\varepsilon_t = 0.0100 > 0.0050$ tension controlled

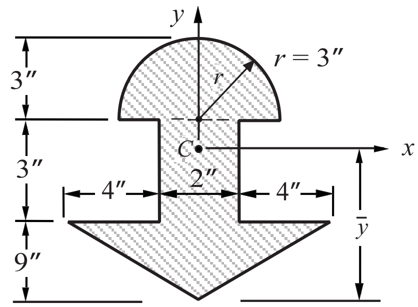
The nominal Moment: (Mn)

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (4.00)(60)\left(24 - \frac{4.71}{2}\right) = 5194.8 \text{ in.-k} = 432.9 \text{ ft-kip}$$

$$\phi M_n = (0.9)(432.9) = 389.6 \text{ ft-kip}$$

$$\boxed{\phi M_n = 389.6 \text{ ft-kip}}$$

Problem: (Centroid / Moments of Inertia)



FE/PE EXAM

The dimensions of a composite area are given as shown in the figure. Using the listed data answer the following questions:

(1) the distance $\bar{y}$ (in.) of the centroid is most nearly

(A) 6.8
(B) 7.0
(C) 7.5
(D) 8.0

(2) the moment of inertia (in.^4) about the horizontal centroidal axis is most nearly (I_{cx})

(A) 725
(B) 793
(C) 826
(D) 952

(3) the moment of inertia (in.^4) about the vertical centroidal axis is most nearly (I_{cy})

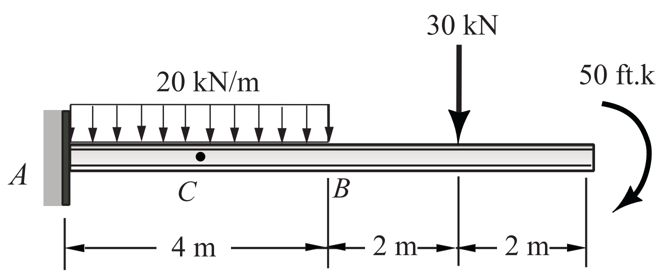
(A) 221
(B) 288
(C) 315
(D) 424

Answers
1- (D)
2- (C)
3- (A)

Important note for students:

This problem is for practice and has four questions. In real FE & PE exams, each problem will have ONLY ONE question.

Problem: (Cantilever Beams)



FE/PE EXAM

Support A : Fixed

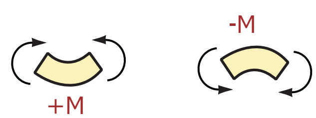
The cantilever beam is loaded as shown in the figure. Using the given loads and the support condition, answer the following questions:

(1) The vertical support reaction (kN) at the fixed support, A_y

(A) 140
(B) 125
(C) 110
(D) 100

(2) The magnitude of the bending moment (kN.m) at the fixed support, M_A

(A) 290
(B) 390
(C) 465
(D) 520



(3) The magnitude of the moment (kN.m) at C, the mid-point of AB, is most nearly, M_C

(A) 515
(B) 420
(C) 338
(D) 210

(4) The shear force (kN) at the mid-point of *AB* is most nearly

(A) 42.5
(B) 50.0
(C) 60.5
(D) 70.0

Important note for students:

This problem is for practice and has four questions. In real FE & PE exams, each problem will have ONLY ONE question.

MCBM-331
ZEYTINCI
SPRNG 2015

FIRST ORDER LINEAR

ORDINARY DIFFERENTIAL EQUATIONS

$$\frac{dy}{dt} = a\,y - b$$

FE EXAM

(a) Find the general solution of this ODE
(b) Find the particular solution when t = 0 then $y = y_o$

Solution:

$$\frac{dy}{dt} = a\,y - b$$

$$\frac{dy}{dt} = a(y - b/a)$$

$$\frac{dy}{(y - b/a)} = a\,dt$$

$$\int \frac{dy}{(y - b/a)} = \int a\,dt$$

$$\ln|\,y - b/a\,| = a\,t + C_1$$

$$\ln|\,y - b/a\,| = a\,t + C_1$$

$$|\,y - b/a\,| = e^{(a\,t + C_1)}$$

$$|\,y - b/a\,| = e^{a\,t} \bullet e^{C_1}$$

$$|\,y - b/a\,| = e^{a\,t} \bullet C_2$$

$$y - b/a = \pm\, C_2\, e^{a\,t}$$

$$\boxed{y = b/a + C\, e^{a\,t}}$$

General Solution

C: Arbitrary constant

Because of the arbitrary constant C we have infinitely many solutions.

Particular solution is also called the **Initial Value Problem:**

$$\left.\begin{matrix} t = 0 \\ y = y_0 \end{matrix}\right\} \quad C = y_0 - {}^{b}/_{a}$$

$$\boxed{y = b/a + (y_0 - {}^{b}/_{a})\, e^{a\,t}}$$

Particular Solution

MATH-640
ZEYTINCI
SPRING 2015

FIRST ORDER LINEAR

ORDINARY DIFFERENTIAL EQUATIONS

$$\frac{dy}{dx} = \frac{1}{2}y - 450$$

FE EXAM

(a) Find the general solution of this ODE

(b) Find the particular solution when $x = 0$ and $y = 850$

Solution:

$$\frac{dy}{dx} = \frac{1}{2}y - 450$$

$$\frac{dy}{dx} = 0.5\,y - 450$$

$$\frac{dy}{dx} = \frac{2\,(0.5\,y - 450)}{2}$$

$$\frac{dy}{dx} = \frac{y - 900}{2}$$

$$\frac{dy}{y-900} = \frac{dx}{2}$$

$$\int \frac{dy}{y-900} = \int \frac{dx}{2}$$

$$\ln|y - 900| = \frac{x}{2} + C_1$$

$$|y - 900| = e^{\frac{x}{2} + C_1}$$

$$|y - 900| = e^{\frac{x}{2}} \cdot e^{C_1}$$

$$|y - 900| = e^{\frac{x}{2}} \cdot C_2$$

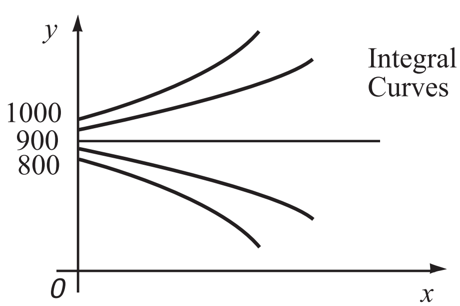
$$y - 900 = \pm C_2\, e^{\frac{x}{2}}$$

$$y = 900 + C\, e^{\frac{x}{2}}$$

General Solution

C: Arbitrary constant

Because of the arbitrary constant *C* there will be infinitely many solutions.



Particular solution is also called
Initial Value Problem:

$$\left| \begin{array}{l} x = 0 \\ y = 850 \end{array} \right\} \quad C = -50$$

$$y = 900 - 50\, e^{x/2}$$

Particular Solution

FIRST ORDER LINEAR
ORDINARY DIFFERENTIAL EQUATIONS

FE EXAM

(1)

$$\frac{dy}{dx} = y - 50$$

The general solution of this First Order Linear Ordinary Differential Equation (ODE) is most nearly:

(A) $y = 200 - C_1 e^{\frac{x}{2}}$

(B) $y = 100 + C_1 e^{\frac{x}{2}}$

(C) $y = C_1 e^{x} + 100$

(D) $y = 50 + C_1 e^{x}$

C_1 : Integration constant

(2)

$$\frac{dy}{dx} = y - x$$

The general solution of this First Order Linear Ordinary Differential Equation (ODE) is most nearly:

(A) $y = x - C_1 e^{\frac{x}{2}}$

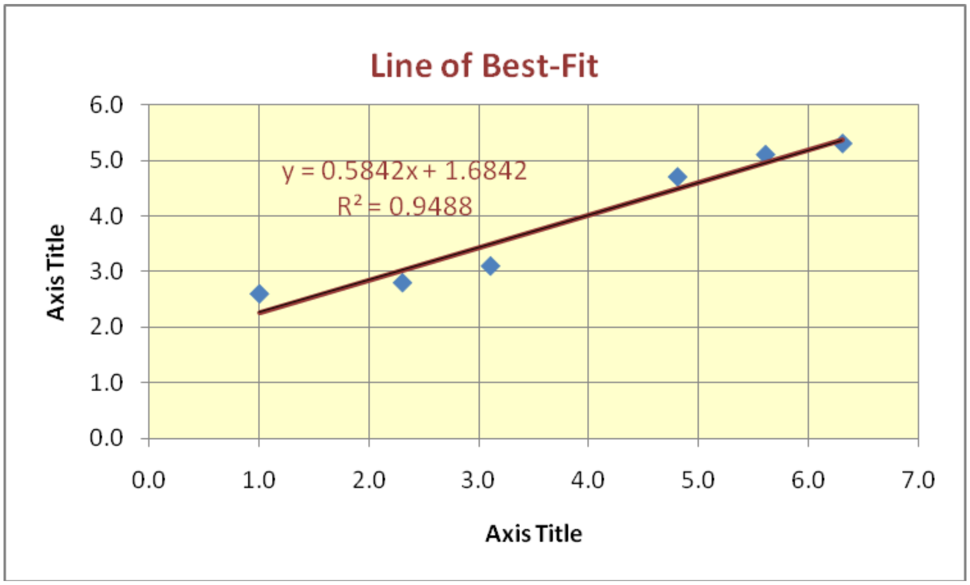
(B) $y = x + e^{-x} + C_1 e^{x}$

(C) $y = x + 1 + C_1 e^{x}$

(D) $y = 50 + C_1 e^{x}$

C_1 : Integration constant

LINEAR REGRESSION EQUATIONS



Equation of the line: $$y = mx + b$$

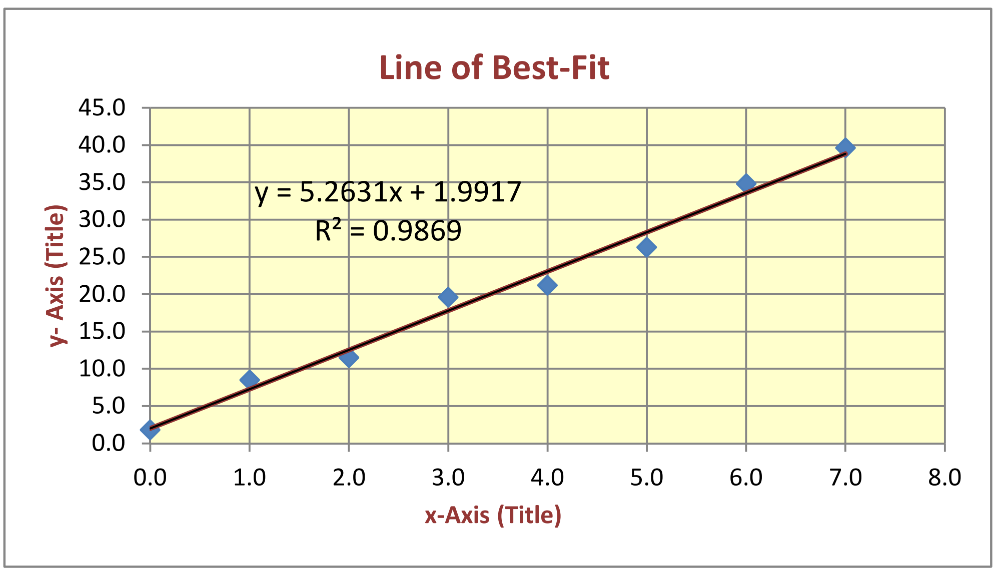
The slope: $$m = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2}$$

The y-intercept: $$b = \frac{\Sigma y_i - m\Sigma x_i}{n}$$

Correlation Coefficient: $$R = \frac{n\Sigma(x_i y_i) - \Sigma x_i \Sigma y_i}{\sqrt{\left[n\Sigma(x_i^2) - (\Sigma x_i)^2\right]\left[n\Sigma(y_i^2) - (\Sigma y_i)^2\right]}}$$

Problem: The Line of Best Fit

	x	y
1	0.0	1.8
2	1.0	8.5
3	2.0	11.5
4	3.0	19.6
5	4.0	21.2
6	5.0	26.3
7	6.0	34.8
8	7.0	39.6



Slope $m = 5.2631$

y-Intercept $b = 1.9917$

Correlation Coeff. R = SQRT (0.9869) = 0.9934

Excellent Correlation!

CASIO / *fx*-115 ES PLUS

MODE	SHIFT SETUP
(1) COMP	(1) Mth-IO
(2) CMPLX	(2) Line-IO
(3) STAT	(3) Deg
(4) BASE-N	(4) Rad
(5) EQN	(5) Gra
(6) MATRIX	(6) Fix
(7) TABLE	(7) Sci
(8) VECTOR	(8) Norm

FE Exams / Two Number Systems:

1- Decimal Number System (base 10)
2- Binary Number System (base 2)

In Decimal System 10 different digits are used to create any number, and in Binary System only 0s and 1s are used to create any number.

DECIMAL	BINARY
2	10
3	11
5	101
6	110
8	1000
9	1001
10	1010
12	1100
14	1110
15	1111
19	10011
25	11001

NUMBER SYSTEMS
BINARY & DECIMAL
NCEES Reference Handbook, Page: 213

Binary Number System:

In digital computers, binary number system (the base-2) is used. Conversions from BINARY to DECIMAL or from DECIMAL to BINARY can easily be done using the calculator. Binary (base-2), decimal (base-10).

Problem:

Find the binary equivalent of decimal 25?

Turn on your calculator

1) Press MODE
2) Press “4”
3) Enter 25 and press “ **=** ”
4) Make sure to see 25 under **Dec** on the screen
5) Press SHIFT then “log”
6) Answer: 11001

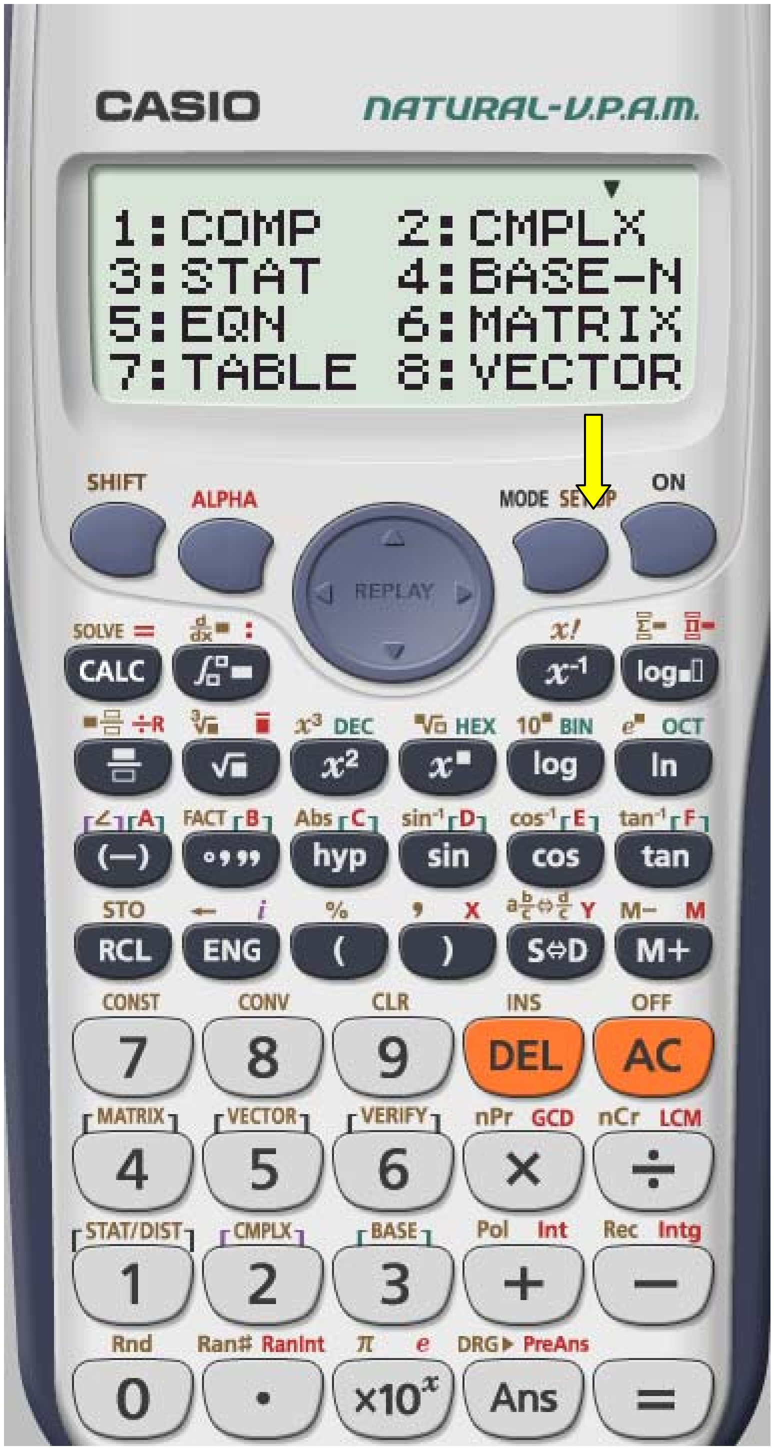
Problem:

Find the decimal equivalent of binary 1111?

Turn on your calculator

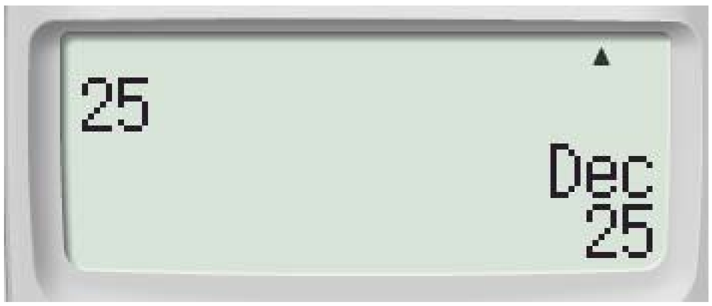
1) Press MODE
2) Press “4”
3) Press SHIFT then press “log” key
4) Enter 1111 and then press “ **=** ”
5) Make sure to see 1111 under **Bin** on the screen
6) Press SHIFT then hit “ x^2 ” key
7) Answer: 15

First Press MODE , then Press 4 for BASE-N



Step-by-step Screen Shots:

Press* [2] [5] *and press* [=] *key



***Press* [SHIFT] *and then* [log]** *to get the answer*

Answer: 11001

Step-by-step Screen Shots:

Press [SHIFT] ***and then*** [log]

Enter [1] [1] [1] [1]

Press [SHIFT] ***then hit*** [x^2] ***to get the answer***

Answer: **15**

CONVERTING BINARY NUMBERS TO DECIMALS

Convert binary 1011 to decimal:

3 2 1 0 ← power of 2

$1011_2 = 1 \times 2^3 \rightarrow 8$

$0 \times 2^2 \rightarrow 0$

$1 \times 2^1 \rightarrow 2$

$1 \times 2^0 \rightarrow 1$

$\rightarrow 11$

Answer: 11

Convert decimal 18 to binary:

$18_{10} = ?$

$18 / 2 = 9$ and rem $= 0$ (_ _ _ _ 0_2)

$9 / 2 = 4$ and rem $= 1$ (_ _ _ 10_2)

$4 / 2 = 2$ and rem $= 0$ (_ _ 010_2)

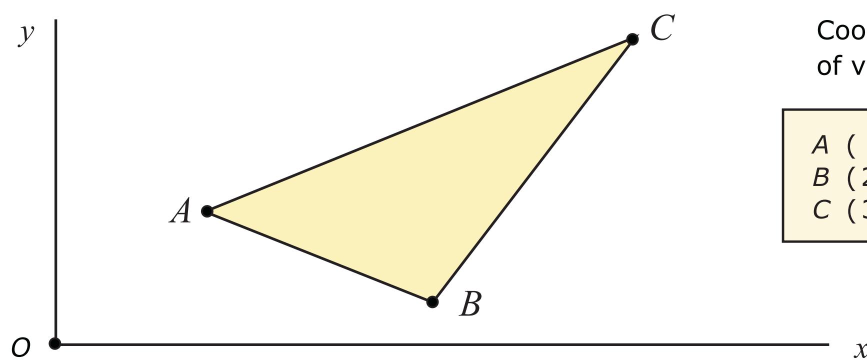
$2 / 2 = 1$ and rem $= 0$ (finish! 10010_2)

(Here keep dividing by base 2)
Until the quotient < base

Answer: 10010

APPLICATIONS OF DETERMINANTS

AREAS OF TRIANGLES



Coordinates of vertices:

A (8, 10)
B (24, 8)
C (30, 17)

A plane triangle is given as shown in the figure. Using the listed coordinates of the vertices *A*, *B*, and *C* answer the following:

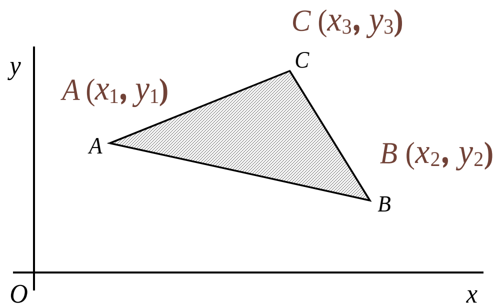
The area (unit ^{2}) of this triangle is most nearly:

(A) 197.5
(B) 125.4
(C) 98.53
(D) 78.00

AREA COMPUTATIONS

Determinants may be conveniently used in *FE* exams when the area computations of triangles with coordinates are required. Simplifying complex mathematical expressions would yield a determinant as shown here:

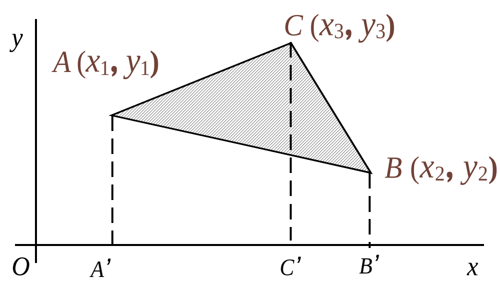
Example:



A triangle *ABC* is given as shown in the figure. Using the listed coordinates of *A*, *B*, and *C* determine the area of the triangle.

Solution:

Draw the vertical dashed lines from points *A*, *B*, and *C*.



Area of the Trapezoid $AA'CC' = \left(\frac{x_3 - x_1}{2}\right)(y_1 + y_3)$

Area of the Trapezoid $BB'CC' = \left(\frac{x_2 - x_3}{2}\right)(y_2 + y_3)$

Area of the Trapezoid $AA'BB' = \left(\frac{x_2 - x_1}{2}\right)(y_2 + y_1)$

Area of *ABC* = Area *AA'CC'*
+ Area *BB'CC'*
- Area *AA' BB'*

This will yield a fairly complex relationship for the area. But if we use determinant relationship, the required area may be calculated from the following simple determinant.

$$\text{Area} = \tfrac{1}{2} \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

ABSOLUTE ERROR & RELATIVE ERROR

The accuracy of a computation is very important in numerical analysis. There are two ways to express the size of the error in a computed result:

(a) Absolute Error
(b) Relative Error

$$\text{ABSOLUTE ERROR} = |\,\text{True Value} - \text{Approximate Value}\,|$$

$$\text{RELATIVE ERROR} = \frac{\text{Absolute Error}}{|\,\text{True Value}\,|}$$

Example:

True Value = 10/3
Approximate Value = 3.333

(a) Determine the absolute error
(b) Determine the relative error
(c) Find the significant digits

Solution:

$$\begin{aligned}\text{ABSOLUTE ERROR} &= |\text{True Value} - \text{Approximate Value}\,| \\ &= 10/3 - 3.333 \\ &= 0.000333... \\ &= 1\,/\,3000\end{aligned}$$

$$\text{RELATIVE ERROR} = \frac{\text{Absolute Error}}{|\text{True Value}\,|}$$

$$= \frac{(\,1/3000)}{(10/3)}$$

$$= \ 1/10{,}000$$

Here, the number of significant digits is 4.